MAT 311 Introduction to Number Theory

Problem Set 1

due Wednesday, February 4

Please prove all your answers. Many (but not all) of the questions below are from Niven–Zuckerman–Montgomery.

Problem 1.
(a) Given $a|b$ and $c|d$, prove that $ac|bd$. Is the converse true?
(b) Given $a|b$ and $a|c$, prove that $a|(bm + cn)$, where b, c are arbitrary integers.

Problem 2. Prove that for every integer n
(a) $n^2 - n$ is divisible by 2
(b) $n^3 - n$ is divisible by 6
(c) $n^2 + 2$ is not divisible by 4

Problem 3. Find values of x and y to satisfy
(a) $423x + 198y = 9$
(b) $71x - 50y = 1$
What conclusions can you make about $gcd(423, 198)$ and $gcd(71, 50)$?

Problem 4. Prove that $(a, a + 2) = 1$ or 2 for every integer a.

Problem 5. Use the Euclidean algorithm (to be discussed Friday) to find $gcd(1109, 4999)$.

Problem 6.
(a) Show that there are no prime triplets, that is primes p, $p + 2$, $p + 4$, other than 3,5, and 7.
(b) Find all prime p such that $p^2 + 1$ is also prime.
(c) Find all prime p such that $p^2 + 2$ is also prime.