MAT 311, Homework 3 due 9/20

1. Prove that 66 divides $43^{23} + 23^{43}$.

2. (a) Find the last two digits of 3^{1000}. **Hint:** use a theorem that gives you a useful congruence for a^n.

(b) Find the last two digits of 2^{1000}. **Hint:** the theorem you used in (a) no longer applies, but you can work around this by factoring $100 = 4 \cdot 25$ and dealing with mod 4 and mod 25 separately.

3. Let R_{71} be the multiplicative group modulo 71, and let $c \in R_{71}$ be the class of 50. Compute the inverse of c in R_{71}.

4. We know that the additive group Z_m modulo m is cyclic for every positive integer m. How many distinct elements of Z_m can serve as the generator? Prove your answer.

Please also do Questions **12, 17, 18, 23** from section 2.1 in the textbook. This homework is based on the material we learned in section 2.1, so please read the textbook!

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(Optional challenge.) Read Lemmas 2.13, 2.14 and Theorem 2.15 in the textbook, and solve Question **38** from 2.1 (the question is based on this additional reading material). Feel free to use the hint given in the textbook.

(Optional challenge.) For p prime, consider a regular p-gon. We want to color its p vertices using a colors (a is a given positive integer). For each coloring, there may be several vertices of the same color, and it's okay to use only some but not all of the colors (in particular, we can use the same color for all of the vertices). How many such colorings are there, up to rotations of the p-gon? ("Up to rotations" means that two colorings are considered to be the same if they look the same after one of them is rotated: for example, there's only one coloring where one vertex is green and $p-1$ vertices are red.)

Use your calculation of the number of colorings to give an alternative proof of Fermat's Little Theorem. Where in your calculation did you use the fact that p is prime?

Optional challenge questions will be graded if you submit them (to give you feedback), but no points will be given.