Reference Page

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$$
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$
$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$$
$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$
$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n} = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots$$

The Taylor series formula for $f(x)$ around a:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

The Taylor inequality for the remainder $R_n = f(x) - \left[f(a) + \frac{f'(a)}{1!}(x-a) + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n\right]$:

$$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1},$$

if $|f^{(n+1)}(x)| \le M$ for all x between a and x.

Approximate integration methods:
Trapezoidal rule:
$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \dots 2f(x_{n-1}) + f(x_n))$;
Midpoint rule:
$\int_a^b f(x)\,dx \approx \Delta x(f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n))$;
Simpson rule (n is even):
$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}(f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) \dots 4f(x_{n-1}) + f(x_n))$.
Error estimates:

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \text{ (trapezoidal rule);}$$
$$|E_M| \le \frac{K(b-a)^3}{24n^2} \text{ (midpoint rule);}$$
$$|E_S| \le \frac{M(b-a)^5}{180n^4} \text{ (Simpson rule).}$$

In the formulas above, the approximation methods are used to evaluate $\int_a^b f(x)\,dx$.
In each method, the interval is divided into n small intervals of length Δx;
$x_0, x_1, \dots x_n$ are the endpoints of the small intervals as usual, $\bar{x}_1, \dots \bar{x}_n$ are the midpoints.
Numbers K, M are such that $|f''(x)| \le K$, $|f^{(4)}(x)| \le M$ for all x in $[a,b]$.

The logistic equation: $\frac{dP}{dt} = kP(1 - \frac{P}{M})$, where M is the carrying capacity