Compliments to Bad Spaces

Oleg Viro

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Inspired by H.Weyl's book on Riemann surfaces Die Idee der Riemannschen Fläche published in 1913.

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not differentiable, but differential,
for nobody is going to differentiate it!

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2. *$f \in \mathcal{C}^r(X)$ if near each point of X it coincides with a function belonging to $\mathcal{C}^r(X)$.*

    In other words, $f \in \mathcal{C}^r(X)$ if for each $a \in X$ there exist $g, h \in \mathcal{C}^r(X)$ such that $h(a) > 0$ and $f(x) = g(x)$ for each x with $h(x) > 0$.

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4. *Topological space.* A topological space X with all continuous functions $X \to \mathbb{R}$.

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Isomorphisms of the category are called *C^r-diffeomorphims*.

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Whitney Problem: Describe the differential structure induced on a closed $X \subset \mathbb{R}^n$.

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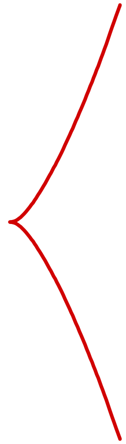

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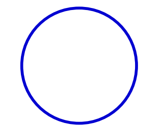
If so, how does the the image look like? Like this:  ?

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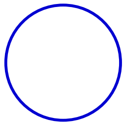
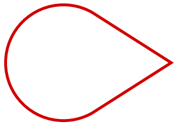
If so, how does the the image look like? Like this:  ?

No, like this:  !

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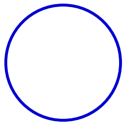
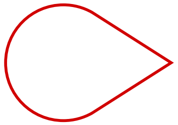
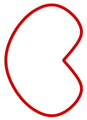
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  !

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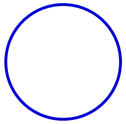
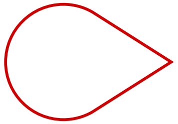
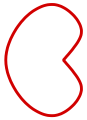
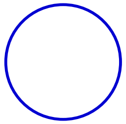
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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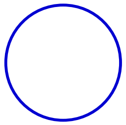
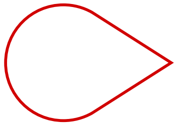
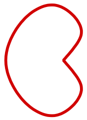
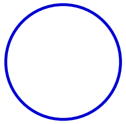
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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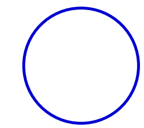
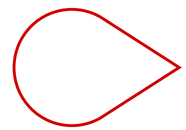
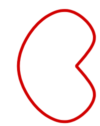
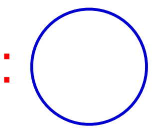
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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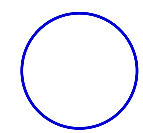
Then we get really a space diffeomorphic to  .

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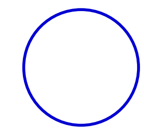
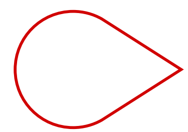
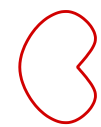
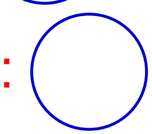
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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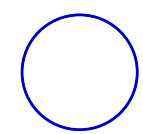
Then we get really a space diffeomorphic to  .

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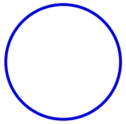
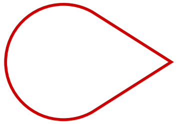
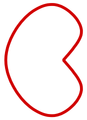
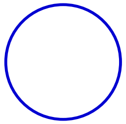
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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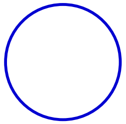
Then we get really a space diffeomorphic to  .

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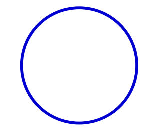
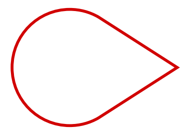
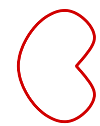
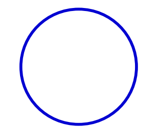
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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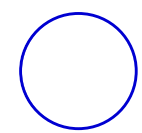
Then we get really a space diffeomorphic to  .

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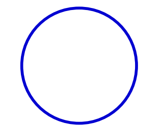
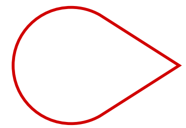
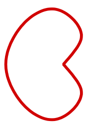
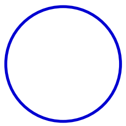
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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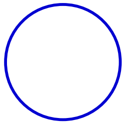
Then we get really a space diffeomorphic to  .

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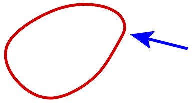
The resulting space:  .

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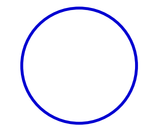
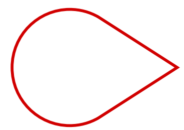
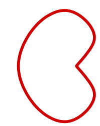
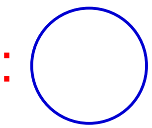
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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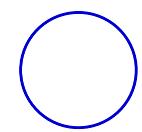
Then we get really a space diffeomorphic to  .

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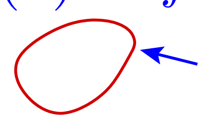
The resulting space:  . Smooth, but with jump of curvature.

Tangent vectors and dimensions

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$$\frac{|f(x) - f(p) - \sum a_i(\mathrm{dist}(q_i, x) - \mathrm{dist}(q_i, p))|}{\mathrm{dist}(x, p)} \to 0$$

as $x \to p$.

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as $x \to p$. Is this definition good? At least, it recovers the smooth structure of a Riemannian manifold.

Finite Topological Spaces

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At early days of topology, they were the main objects of the *Combinatorial Topology*.

Fundamental group

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Especially if the partition was a triangulation.

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The star $St(\sigma)$ of a face σ is the union of all faces Σ such that $\partial\Sigma \supset \sigma$.

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P can be recovered from Q.

Homotopy

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Corollary. *All homotopy and singular homology groups of P and Q are isomorphic.*

Corollary. *Any compact polyhedron is weak homotopy equivalent to a finite topological space.*

Digital plane and Jordan Theorem

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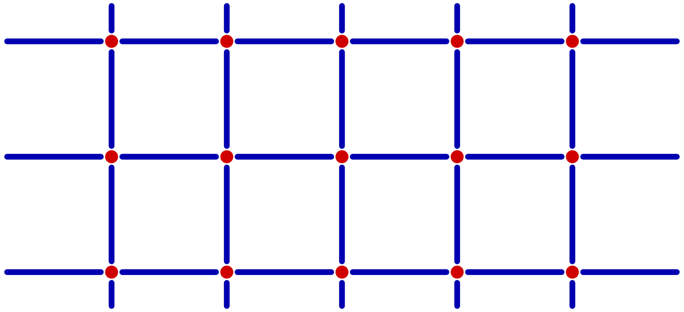

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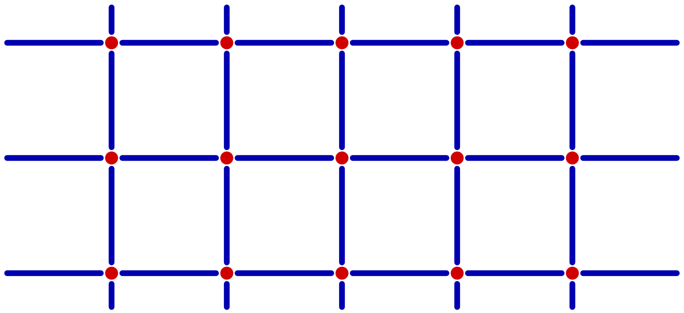

Digital circle of length d is the quotient space of the circle $S^1 \subset \mathbb{C}$ by the partition formed by complex roots of unity of degree d and open arcs connecting the roots next to each other.

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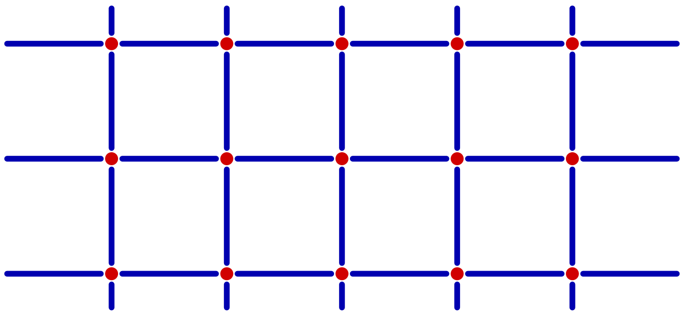
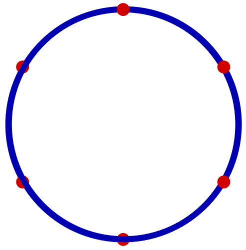


Digital circle



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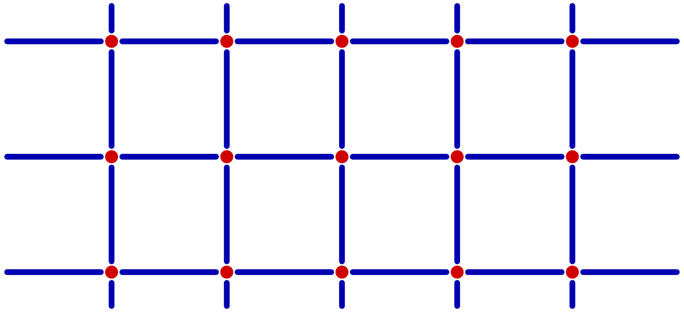

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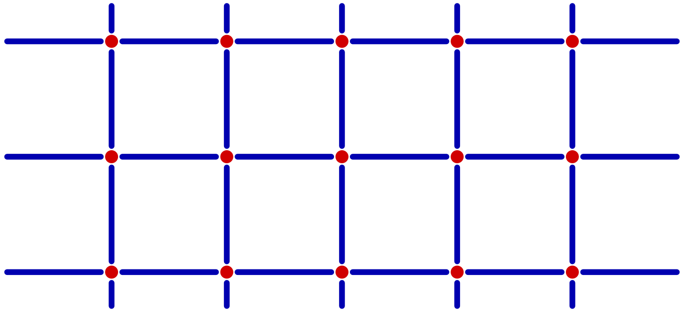

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Not finite, but *locally finite*.

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The quotient space by the T_0-equivalence relation satisfies the Kolmogorov separation axiom T_0:
for any pair of points x, y *at least one of them has a neighborhood not containing the other one.*

Arbitrary finite space

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(if x and y are T_0-equivalent, then both $x \in \operatorname{Cl} y$ and $y \in \operatorname{Cl} x$).

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Baricentric subdivision

How far is a poset topology from the face space of a polyhedron?

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Not really far, just one step construction.

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Theorem. *Any finite topological space is weak homotopy equivalent to a compact polyhedron.*