SOME INTEGRAL CALCULUS BASED ON EULER CHARACTERISTIC

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Sometimes in appears to be useful to consider the Euler characteristic as a (finitely-additive) measure and, in particular, to integrate with respect to it. The following notes are gathered to justify this point of view.

GENERALITIES

1. Integration with respect to a finitely-additive measure.

Let X be a set, $\mathfrak{A}$ a collection of subsets of X closed with respect to be operations of (finite) union and (finite) intersection. Let R be a commutative ring and $\mu : \mathfrak{A} \longrightarrow R$ a function with the property

$$\mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B)$$

for $A, B \in \mathfrak{A}$. Let $\mathcal{F}(X, \mathfrak{A}, \mu)$ denote the ring of finite R-linear combinations of characteristic functions of elements of $\mathfrak{A}$. It is not difficult to prove the following assertion.

1A. For any $B \in \mathfrak{A}$ there is a well defined functional

$$\mathcal{F}(X,\mathfrak{A},\mu) \longrightarrow R : f \longmapsto \int_B f(x)\, d\mu(x) \quad \text{with} \quad \int_B f(x)\, d\mu(x) = \sum \lambda_A \mu(A \cap B) \ \text{ if } \ f = \sum \lambda_A \mathbb{1}_A .$$

2. Integration with respect to Euler characteristic.

Below the construction of Section 1 is applied in the following more specialized situations: X is a topological space, each element $A \in \mathcal{A}$ has a well defined Euler characteristic $\chi(A)$. Below elements of $\mathcal{A}$ are called t a m e s e t s . We take $R = \mathbb{Z}$ and $\mu = \chi$ and abbreviate $\mathcal{F}(X, \mathcal{A}, \mathbb{Z})$ to $\mathcal{F}(X)$. Thus on $\mathcal{F}(X)$ we have a well defined integration-operation which assigns to a function $f = \sum_{A \in \mathcal{A}} \lambda_A \mathbb{1}_A$ and a set $B \in \mathcal{A}$ the number

$$\int_B f(x)\, d\chi(x) = \sum \lambda_A \chi(A \cap B)$$

The basic example of such a situation: X a projective algebraic variety over $\mathbb{R}$ or $\mathbb{C}$, $\mathcal{A}$ an algebra of closed semi-algebraic sets.

3. Fubini theorem and Riemann-Hurwitz theorem.

Another fundamental property of the Euler characteristic is its multiplicativity: $\chi(X \times Y) = \chi(X)\chi(Y)$. It implies that if $E \to B$ is a locally-trivial fibration with fibre F then $\chi(E) = \chi(B)\chi(F)$

To use and extend this property let us introduce the following notion. Let X and Y be spaces with algebras $\mathcal{A}$ and $\mathcal{B}$ of tame sets. A map $\varphi: X \to Y$ is said to be t a m e (with respect to $\mathcal{A}$ and $\mathcal{B}$) if (i) $\varphi^{-1}(y) \in \mathcal{A}$ for any $y \in Y$ and (ii) for any $A \in \mathcal{A}$ there exist $S_0 \subset S_1 \subset \ldots$ with $S_i \in \mathcal{B}$ such that the maps

$$\varphi^{-1}(S_i \setminus S_{i-1}) \cap A \longrightarrow S_i \setminus S_{i-1}$$

defined by φ are locally-trivial fibrations.

It is well known that any regular map of a real algebraic variety to another one is tame with respect to the algebras of closed semi-algebraic sets.

The following theorem obviously follows from the multiplicativity property of χ stated above and extends it.

3.A. (Fubini-type theorem). If $\varphi: X \to Y$ is a tame map and $f \in \mathcal{F}(X)$ then

$$\int_X f(x)\, d\chi(x) = \int_Y \left(\int_{\varphi^{-1}(y)} f(x)\, d\chi(x) \right) d\chi(y) .$$

The following corollary of 3.A is a generalization of the Riemann-Hurwitz formula (the well known relation connecting Euler characteristics of the covering space, the base and the branch indeces of a branched covering).

3.B. Let $\varphi : X \longrightarrow Y$ be a tame map and $b \in \mathcal{F}$ a function such that $\int_{\varphi^{-1}(y)} b(x)\, d\chi(x)$ does not depend on $y \in Y$. Put $d = \int_{\varphi^{-1}(y)} b(x)\, d\chi(x)$. Then

$$\chi(Y)\, d = \int_X b(x)\, d\chi(x)$$

or, equivalently,

$$\chi(X) = \chi(Y)\, d - \int_X (b(x) - 1)\, d\chi(x)$$

If X and Y are surfaces, φ a branched covering, d the degree of φ and $b(x)$ the branch index at $x \in X$, then the latter relation is just the Riemann-Hurwitz formula.

For a fixed $\varphi : X \longrightarrow Y$ the functions $b \in \mathcal{F}(X)$ satisfying the hypothesis of 3.B constitute a subgroup of $\mathcal{F}(X)$, which can be rather vast, but in some cases there is naturally distinguished b. For example if X and Y are complex varieties, Y is irreducible, and φ is homomorphic, then for $b(x)$ one can get the Euler characteristic of $\mathcal{U} \cap \varphi^{-1}(y)$ where $\mathcal{U}$ is a regular neighbourhood of x in X and y is a generic point of Y sufficiently close to $\varphi(x)$.

INTEGRAL GEOMETRY

4. Radon-type transformation.

In this section "tame set" means "closed semi-algebraic set". Here we consider a Radon-type transformation with respect to Euler characteristic for the n-dimensional complex projective space $\mathbb{C}P^n$.

Let $\mathbb{C}P^{n\vee}$ denote the dual (to $\mathbb{C}P^n$) space consisting of hyperplanes of $\mathbb{C}P^n$

The Radon-type transformation is the map $\mathcal{F}(\mathbb{C}P^n) \longrightarrow \mathcal{F}(\mathbb{C}P^{n\vee})$ which assigns the function $f^\vee : H \longmapsto \int_H f(x)\, d\chi(x)$ to a function $f \in \mathcal{F}(\mathbb{C}P^n)$ It is easy to verify that

$$f^{\vee\vee}(x) = f(x) + (n-1)\int_{\mathbb{C}P^n} f(x)\, d\chi(x) \tag{1}$$

Let $\mathcal{G}(X)$ denote the factor-group $\mathcal{F}(X)/\mathbb{Z}$, where $\mathbb{Z}$ is realized in $\mathcal{F}(X)$ as the subgroup of constant functions. It follows from (1) that the map $\mathcal{G}(\mathbb{C}P^n) \longrightarrow \mathcal{G}(\mathbb{C}P^{n\vee})$ induced by the Radon-type transformation $\mathcal{F}(\mathbb{C}P^n) \longrightarrow \mathcal{F}(\mathbb{C}P^{n\vee})$ is a duality.

The definition of the transformation $f \longmapsto f^\vee$ does not involve the complex conjigation $\mathbb{C}P^n \longrightarrow \mathbb{C}P^n$, but this transformation has the following useful r e a l property, that is fairly unexpected since the Radon-type transformations with respect to the usual measures have no analogous property.

<u>4.A.</u> For any $f \in \mathcal{F}(\mathbb{C}P^n)$

$$\int_{\mathbb{R}P^{n\vee}} f^\vee(x)\, d\chi(x) = \int_{\mathbb{R}P^n} f(x)\, d\chi(x) \qquad \text{if } n \text{ is odd },$$

$$\int_{\mathbb{R}P^{n\vee}} f^\vee(x)\, d\chi(x) = \int_{\mathbb{C}P^n \setminus \mathbb{R}P^n} f(x)\, d\chi(x) \qquad \text{if } n \text{ is even }.$$

PROOF. Let $X = \{(x,y) \in \mathbb{R}P^{n\vee} \times \mathbb{C}P^n \mid y \in x\}$. By the definition of $f^\vee$,

$$\int_{\mathbb{R}P^{n\vee}} f^\vee(x)\, d\chi(x) = \int_{\mathbb{R}P^{n\vee}} \int_x f(y)\, d\chi(y)\, d\chi(x).$$

By 3.A,

$$\int_{\mathbb{R}P^{n\vee}} \int_x f(y)\, d\chi(y)\, d\chi(x) = \int_X f(y)\, d\chi(x,y) =$$

$$= \int_{\mathbb{C}P^n} \Big(\int_{\{x \in \mathbb{R}P^{n\vee} \mid y \in x\}} f(y)\, d\chi(x) \Big)\, d\chi(y) =$$

$$= \int_{\mathbb{C}P^n} f(y)\, \chi\{x \in \mathbb{R}P^{n\vee} \mid y \in x\}\, d\chi(y).$$

Since $\{x \in \mathbb{R}P^{n\vee} \mid y \in x\}$ is homeomorphic to $\mathbb{R}P^{n-1}$, if $y \in \mathbb{R}P^n$, and to $\mathbb{R}P^{n-2}$, if $y \in \mathbb{C}P^n \setminus \mathbb{R}P^n$, the evaluation of $\chi\{x \in \mathbb{R}P^n \mid y \in x\}$ completes the proof. $\square$

5. Dual projective varieties.

Let $A \in \mathbb{C}P^n$ be a variety. Its dual variety $A^\vee \subset \mathbb{C}P^n$ is the closure of the set of hyperplanes tangent to A at its non-singular points. It is really a duality in the sense that $A^{\vee\vee} = A$. Here we consider a connection between this duality and the duality $\mathcal{G}(\mathbb{C}P^n) \to \mathcal{G}(\mathbb{C}P^{n\vee})$ appeared above in Section 4.

In this section "tame set" means "Zariski closed set". This choice of the algebra of tame sets implies that any $f \in \mathcal{F}(\mathbb{C}P^n)$ has a well defined generic value, i.e. the number v such that $f(x) - v$ vanishes on the complement of some variety of dimension $< n$. Denote $f(x) - v$ by $\overline{f(x)}$. The correspondence $f \longmapsto \overline{f}$ induces the inclusion $\mathcal{G}(\mathbb{C}P^n) \to \mathcal{F}(\mathbb{C}P^n)$ which is left inverse to the natural projection $\mathcal{F}(\mathbb{C}P^n) \to \mathcal{G}(\mathbb{C}P^n)$.

It is not difficult to prove the following theorem.

5.A. For any variety $A \in \mathbb{C}P^n$ with $A \neq \emptyset$ there exists $f \in \mathcal{F}(\mathbb{C}P^n)$ such that $A \supset \operatorname{supp} f \neq \emptyset$ and $A^\vee \supset \operatorname{supp}(\overline{f^\vee})$. Such f is unique up to constant factor.

The functions f of 5.A are constant on the set of nonsingular points of A. Thus there is unique of these functions that equals 1 on nonsingular points of A. We denote it by f_A. Theorem 5.A implies that $\overline{f_A^\vee} = \pm f_{A^\vee}$

5.B. Conjecture. $f_A(x)$ is equal to MacPherson's local Euler obstruction $Eu_x(A)$ defined in [6].

6. Dual curves.

Let A be a complex plane projective curve. For a point $x \in \mathbb{C}P^2$

let $m_A(x)$ denote the intersection number at x of A and a generic line passing through x.

6.A. For any $y \in \mathbb{C}P^{2\vee}$

$$m_{A^\vee}(y) = \deg A - (m_A)^\vee(y) \quad (= \deg A - \int_y m_A(x)\, d\chi(x).$$

This formula can be proved by applying the Riemann-Hurwitz formula to a projection of a regular neighbourhood of $A \cap y$ from a generic point $p \in \mathbb{C}P^2$ lying sufficiently close to y. It has the following well known local variant.

6.B. Let $x \in \mathbb{C}P^2$ lie on only one branch of A and $y \in \mathbb{C}P^{2\vee}$ lie on only one branch of $A^\vee$. Then

$$m_A(x) + m_{A^\vee}(y) = A \underset{x}{\circ} y = A^\vee \underset{y}{\circ} x$$

The relation 6.A shows that $f_A = m_A$ for a plane projective curve A. Integrating the relation 6.A over a generic line lying on $\mathbb{C}P^{2\vee}$ (i.e. over a generic pencil of lines of $\mathbb{C}P^2$) gives one of the Plucker formulae:

6.C $$\deg A^\vee = 2 \deg A - \int_A m_A(x)\, d\chi(x).$$

6.D. (Generalized Klein formula). For any complex plane projective curve A (which is not necessarily real)

$$\deg A - \int_{A \cap \mathbb{R}P^2} m_A(x)\, d\chi(x) = \deg A^\vee - \int_{A^\vee \cap \mathbb{R}P^{2\vee}} m_{A^\vee}(x)\, d\chi(x).$$

PROOF. By 6.A, $(m_A)^\vee(x) = \deg A - m_{A^\vee}(x)$. Therefore by 4.A,

$$\int_{\mathbb{R}P^{2\vee}} [\deg A - m_{A^\vee}(x)]\, d\chi(x) = \int_{\mathbb{C}P^2 \setminus \mathbb{R}P^2} m_A(x)\, d\chi(x).$$

Thus $\deg A - \int_{\mathbb{R}P^{2\vee}} m_{A^\vee}(x)\, d\chi(x) = \int_{\mathbb{C}P^2} m_A(x)\, d\chi(x) - \int_{\mathbb{R}P^2} m_A(x)\, d\chi(x)$

By 6.C $\int_{\mathbb{C}P^2} m_A(x)\, d\chi(x) = 2 \deg A - \deg A^\vee$. These two equalities imply the desired result. $\square$

Theorem 6.D for the case of real A, which as well as $A^\vee$ has only simple nodes and cusps as singularities, was found by F.Klein [5]

For the case of arbitrary real A it was proved (and stated without integrals with respect to Euler characteristic) by F.Schuh [8] .

7. Dual surfaces.

For a complex surface $A \subset \mathbb{C}P^3$ and $x \in \mathbb{C}P^3$ let $m_A(x)$ denote the intersection number at x of A and a generic line passing through x , and $\mu_A^{(2)}(x)$ the Milnor number at x of the section of A by a generic plane passing through x .

The function f_A defined in Section 5 can be described in the case of a complex surface $A \subset \mathbb{C}P^3$ as follows.

7.A. If $\mathcal{U}$ is a small ball centred at $x \in A$ and P is a generic plane sufficiently close to x , then

$$f_A(x) = \int_{\mathcal{U} \cap A \cap P} m_A(x)\, d\chi(x) \tag{2}$$

and

$$f_{A^\vee}(x) = (f_A)^\vee(x) - e \quad \left(= -e + \int_x f_A(y)\, d\chi(y)\right) \quad , \tag{3}$$

where e is the Euler characteristic of a generic plane section of A.

The f_A can be described in more standard terms too:

7.B. Let $A \subset \mathbb{C}P^3$ be a complex surface, $C_1, \ldots, C_r$ be all the irreducible curves contained in the set of singular points of A . Let m_i denote a generic value of $m_A(x)$ for $x \in C_i$ and μ_i a generic value of $\mu_A^{(2)}(x)$ for $x \in C_i$. Let $m_{C_i}(x)$ denote the intersection number at x of C_i and a generic plane passing through x . Then for any $x \in A$

$$f_A(x) = 1 - \mu_A^{(2)}(x) + \sum_{i=1}^{r} (m_i + \mu_i - 1)\, m_{C_i}(x) .$$

Integrating the relation (3) over a generic line and a generic plane in $\mathbb{C}P^{3\vee}$ gives the following Plücker-type formulae.

7.C. If e is Euler characteristic of a generic plane section of a complex surface $A \subset \mathbb{C}P^3$ and e is Euler characteristic of a generic plane section of $A^\vee$, then

$$\int_A f_A(x)\, d\chi(x) = \deg A^\vee - \deg A + 2e, \tag{4}$$

$$e - 2\deg A = e^\vee - 2\deg A^\vee . \tag{5}$$

Relations (3) and Theorem 4.A imply the following theorem.

7.D. (Klein-type formula). For any complex surface $A \subset \mathbb{C}P^3$ (which is not necessarily real)

$$\int_{A\cap \mathbb{R}P^3} f_A(x)\, d\chi(x) = \int_{A^\vee \cap \mathbb{R}P^{3\vee}} f_{A^\vee}(x)\, d\chi(x) .$$

8. Plane algebraic wave fronts.

Let C_r denote the space of plane circles of radius r . Since a circle of radius r is defined by the equation

$$x^2 + y^2 - 2ux - 2vy + w = 0$$

with $w = u^2 + v^2 - r^2$, the C_r is the paraboloid defined in the 3-space with coordinates u, v, w by the equation $u^2 + v^2 - w - r^2 = 0$.

Let A be a plane affine real algebraic curve. The curve on C_r that consists of the circles tangent to A is denoted by $\Phi_r(A)$ and called the wave front. The same term denotes also the image of $\Phi_r(A)$ under the natural projection $C_r \to \mathbb{R}^2 : (u,v,w) \mapsto (u,v)$. For the sake of simplicity suppose that the set $\mathbb{C}A$ of complex points of the projective closure of A does not contain the points $(0:1:\pm i)$ (the circular points). Arguments similar to the proof of 6.A give the following relation.

8.A. For any $y \in C_r$

$$m_{\Phi_r(A)}(y) = 2\deg A - \int_{C_y} m_A(x)\, d\chi(x) .$$

By integrating this relation for $y \in \Phi_r(A)$ with respect to χ and using 6.D one can obtain to following relation announced in my note [10] .

8.B. $\int_{\Phi_\tau(A)} m_{\Phi_\tau(A)}(y)\,d\chi(y) = 2\int_{\mathbb{R}A^\vee} m_{A^\vee}(x)\,d\chi(x) + 2\int_{\mathbb{C}A\cap(\mathbb{C}P^1_\infty\setminus\mathbb{R}P^1_\infty)} m_A(x)\,d\chi(x)$
where $\mathbb{C}P^1_\infty$ and $\mathbb{R}P^1_\infty$ are the complex and real lines of infinite points.

In particular, if $\mathbb{C}A \not\ni (0:1:\pm i)$,then $\int_{\Phi_\tau(A)} m_{\Phi_\tau(A)}(y)\,d\chi(y)$ does not depend on τ . Thus we have a kind of conservation law. The condition $\mathbb{C}A \not\ni (0:1:\pm i)$ is essential, as the case of A= circle shows.

MISCELLANEOUS

9. Rohlin's formula on the complex orientation of a real curve rewritten.

Let $A \subset \mathbb{R}P^2$ be a real non-singular plane projective algebraic curve of an even degree $2k$. Suppose A halves its complexification $\mathbb{C}A \subset \mathbb{C}P^2$. Then the halves, as oriented surfaces (with the orientation determined by the complex structure) induce two opposite orientations of A , since A is their common boundary. These orientations of A are called the complex orientations of A . V.A.Rohlin defined them and found a fundamental restriction on them [7] . Here I rewrite the restriction in terms of integration with respect to Euler characteristic. It has been done essentially by Sharp [9] .

Each component of A devides $\mathbb{R}P^2$ into two parts. One of them is homeomorphic to disk, it is called the inner side, the other is homeomorphic to Möbius band, it is called the outer side. The complex orientation of A determines the orientation of the inner side of a component of A . We fix the orientation of the union of inner sides of components of A via the outer components (i.e. components which do not lie in inner sides of other components). Each point of these union recieves then the index with respect to A . It equals the difference between the number of components of A which encircle this point and determine the same orientation at it (as the outer component encircling it does) and the number of the other components which encircle the point. A point that lies in inner side of no component of A receives index zero. The index of $x \in \mathbb{R}P^2$ (with respect to A) is denoted by $i_A(x)$

9.A. (Rohlin's formula)

$$\int_{\mathbb{R}P^2} [\, i_A(x)\,]^2 \, d\chi(x) = k^2 \, .$$

10. Groemer's generalization of mixed volumes.

In this section "tame set" means "union of a finite collection of compact convex subsets of $\mathbb{R}^n$ ". The fundamental role in the subject under consideration is played by the following observation of H.Groemer [4] .

10.A. If A and B are compact convex subsets of $\mathbb{R}^n$, then

$$\mathbb{1}_{A+B} = \mathbb{1}_A \underset{\chi}{*} \mathbb{1}_B \, ,$$

where $A + B = \{ x + y \mid x \in A, y \in B \}$ is the usual Minkowski sum and $\underset{\chi}{*}$ denotes the convolution transformation with respect to Euler characteristic

$$\mathcal{F}(\mathbb{R}^n) \times \mathcal{F}(\mathbb{R}^n) \longrightarrow \mathcal{F}(\mathbb{R}^n) : (f, g) \longmapsto f \underset{\chi}{*} g$$

with $(f \underset{\chi}{*} g)(x) = \int_{\mathbb{R}^n} f(x-y)\, g(y)\, d\chi(y)$.

In [4] Groemer does not use the symbol of integral and the word convolution. Explicitely this interpretation was pointed out by V.P. Fedotov [2, Chapter 4] . Theorem 10.A suggests to substitute the convolution $\underset{\chi}{*}$ for the Minkowski sum in the definition of mixed volumes. It takes the opportunity of extending this definition to the case of non-convex sets. In fact for any $A_1, \dots, A_n$, which are finite unions of compact convex sets, the function

$$(\lambda_1, \dots, \lambda_n) \longmapsto \int_{\mathbb{R}^n} (\mathbb{1}_{\lambda_1 A_1} \underset{\chi}{*} \dots \underset{\chi}{*} \mathbb{1}_{\lambda_n A_n})(x)\, dx$$

is a homogeneous polynomial (here the integral is usual, with respect to the Lebesgue measure). The coefficients of this polynomial are the mixed volumes of $A_1, \ldots, A_r$ (up to constant factors). See Groemer [4] One can find there also some curious relations which involve integration with respect to Euler charactestic and usual Lebesgue measure.

11. d-to-1 maps.

Purely topological applications of integration with respect to are limited since there are continuous and even smooth maps, which are non-tame with respect to any reasonable algebras of tame sets. However in the PL -category each space has the natural algebra of tame sets namely, algebra of compact PL -subsets, and any PL-map is tame with respect to these algebras. Consequently some problems, which seem to be highly non-trivial in Top-and Diff-categories, are trivial in PL For example, such a problem is the old problem on d -to-1 maps. In its simplest form it asks whether there exists a d -to-1 map $\mathcal{D}^n \to \mathcal{D}^n$ See Chernavskiĭ [3] .

11.A. Let X and Y are spaces with some algebras of tame sets. If there exists a tame d-to-1 map $X \to Y$ then $\chi(X) = d\chi(Y)$

11.A follows from 3.B for in this situation one can put $\ell(x) = 1$.$\square$

11.B. (corollary). There does not exist a d -to-1 map $\mathcal{D}^n \to \mathcal{D}^n$ with $d > 1$, which is tame with respect to any algebras of tame sets.

12. Concluding remarks.

The examples described above seems to show that integration with respect to Euler characteristic can be useful from the heuristic point of view as well as for making statements more compact and expressive as well as a technique of proofs. It is closely related with the sheaf techniques. For example the Fubini-type theorem 3.A is related to the construction of direct image of a sheaf, etc. Compare, for example, [1] . However the integration with respect to Euler characteristic is much more elementary and sometimes easier to apply.

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Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

Subseries: USSR
Adviser: L. D. Faddeev, Leningrad

1346

O. Ya. Viro (Ed.)

Topology and Geometry – Rohlin Seminar

Springer-Verlag
Berlin Heidelberg New York London Paris Tokyo

Editor

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Fontanka 27, 191011 Leningrad, D-11, USSR

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198904 Leningrad, USSR

Mathematics Subject Classification (1980): 57R, 57N, 57M, 55Q, 14H, 14J, 14M10, 05B, 52A25

ISBN 3-540-50237-8 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-50237-8 Springer-Verlag New York Berlin Heidelberg

Printed in Germany

Printing and binding: Druckhaus Beltz, Hemsbach/Bergstr.
2146/3140-543210