

## Final Exam

Examination time: 2:15-4:45pm. No electronic devices, books or notes. Show all your work. Answers without solutions will give no credit.

Name \_\_\_\_\_

Student ID # \_\_\_\_\_

Problem #	Points/total
1	/10
2	/5
3	/15
4	/5
5	/5
6	/5
7	/5
<b>Total</b>	<b>/100</b>

Name \_\_\_\_\_

**Problem 1. (10 pt)** Formulate and prove the theorem about the center of circle circumscribed about a triangle and the theorem about concurrence of altitudes in a triangle.

Name \_\_\_\_\_

**Problem 2. (5 pt)** Construct a triangle  $ABC$ , given an angle congruent to its interior angle at vertex  $A$ , a segment congruent to the median  $BM$ , and a segment congruent to the side  $AB$ .

Name \_\_\_\_\_

**Problem 3. (15 pt)** (a) Formulate necessary and sufficient conditions for two lines to be parallel in terms of same-side interior angles formed by each of the lines and a line transversal to them.

(b) Formulate the Euclidean parallel axiom.

(c) What is the relation between the Euclidean parallel axiom and the condition for parallel lines considered in the part (a) of this problem? Is the axiom necessary

- in the proof of necessity,
- in the proof of sufficiency?

Name \_\_\_\_\_

**Problem 4. (5 pt)** Let  $ABC$  be a triangle, and  $c$  be a circle tangent to the side  $BC$  and lines extending the sides  $AB$  and  $AC$ . Find the degree of  $A$  with respect to  $c$ , if the lengths of sides  $a = |BC|$ ,  $b = |AC|$  and  $c = |AB|$  are known.

Name \_\_\_\_\_

**Problem 5. (5 pt)** Let chords  $AB$  and  $CD$  of the same circle  $q$  bisect each other (i.e., their intersection point  $E$  is the midpoint for both  $AB$  and  $CD$ ). Does it follow that  $|AB|$  and  $|CD|$  are diameters of  $q$ ? Justify your answer with a proof.

Name \_\_\_\_\_

**Problem 6. (5 pt)** Given a circle  $c$  and a point  $A$  outside of  $c$ , find a line passing through  $A$  and meeting  $c$  at points  $P$  and  $Q$  such that  $|AP| = |PQ|$ .

Name \_\_\_\_\_

**Problem 7. (5 pt)** Two lines orthogonal to each other pass through the center of a square. Prove that the segments of the lines cut by sides of the square are congruent.