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Geometric Structures
MAT 360
February 18, 2010

Lecture 7.

Solution of a homework problem on the sum of distances to the sides of a regular triangle.

This problem has a short and nice solution based on areas: the regular triangle ABC is subdivided into three triangles whose bases are the sides AB, BC and CD of the regular triangle and vertex is a point D. The sum of areas of these triangles is the semiperimeter of $\triangle ABC$ multiplied by the sum of distances from D to the sides. On the other hand, the sum of areas is nothing but the area of $\triangle ABC$, and hence it does not depend on D.

Observe, that regularity of the triangle is not used in the solution above. This condition was added to simplify the problem for a student who does not know about areas.

Without areas the problem can be solved as follows.

Lemma 1. For any point D in the regular triangle ABC and a point E on the perpendicular dropped from D to a side of $\triangle ABC$, the sums of distances are the same.

Lemma 2. Any two points inside $\triangle ABC$ can be connected by a broken line of two edges each of which is perpendicular to a side of $\triangle ABC$.

In the preceding lecture we started to work on the following Problem.

Problem 1. Given a circle c, lines l and m and a segment s, construct a segment AB parallel to m, congruent to s with $A \in c$ and $B \in l$.

It was not solved then and solved in this lecture. The idea of solution was to forget for a while one of the conditions, the requirement $A \in c$. Then the number of solutions becomes infinite. The locus of points A consists of two lines parallel to l. Then taking into account the requirement $A \in c$, we have to consider the intersection points of these two lines with c.

For some data the intersection may be empty, for others the number of intersection points (and the solutions of Problem 1) varies.

A solution of a construction problem should consist of the following 4 stages:

(1) Analysis (assume that the required figure has been constructed, construct auxiliary figures, study their properties.).
(2) Construction (describe the required construction as an algorithm: what constructions should be done first, second, etc.).
(3) Proof (prove that the construction gives the figure with the desired properties, that is prove the properties).
(4) Research (investigate, under what conditions the desired figure exists, how the number of figures with the desired properties exist for data in different ranges, etc.).

Problem 2 (Kiselev 102). Construct a quadrilateral $ABCD$ given segments congruent to its sides and to the segment connecting the midpoints of sides AB and CD.

Please, read details in the textbook.