Stony Brook University
Mathematics Department
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Advanced Topology, Geometry I
MAT 540
September 18, 2020

Homework 4

1. (6 pt)
(a) Prove that if (X, A) is a Borsuk pair, then $(X \times \{0\}) \cup (A \times I)$ is a retract of $X \times I$.

(b) Prove that if A is a closed set in X and $(X \times \{0\}) \cup (A \times I)$ is a retract of $X \times I$, then (X, A) is a Borsuk pair.

(c) If X is a Hausdorff space and $A \subset X$, then (X, A) is a Borsuk pair if and only if $(X \times \{0\}) \cup (A \times I)$ is a retract of $X \times I$.

2. (5 pt) Prove that if (X, A) is a Borsuk pair, and the inclusion $A \to X$ is a homotopy equivalence, then A is a deformation retract of X.

3. (5 pt) Prove that if A is a deformation retract of X and

$$(X \times \{I\}, (X \times \{O\}) \cup (A \times I) \cup (X \times \{1\})$$

is a Borsuk pair, then A is a strong deformation retract of X.

4. (4 pt) Construct a pair (X, A) which is not a Borsuk pair with A consisting of one point.

5. (5 pt) Let (Y, A) be a Borsuk pair, A be closed $f : X \to X'$ be a homotopy equivalence, and $\varphi : A \to X$ be a continuous map. Prove that then the map $X \cup_\varphi Y \to X' \cup_{f \circ \varphi} Y$ determined by id_Y and f is a homotopy equivalence.