Stony Brook University
Mathematics Department
Oleg Viro

Advanced Topology, Geometry I
MAT 540
September 11, 2020

Homework 3

1. Let X, Y and Z be sets. To each map $\varphi : X \times Y \to Z$ there corresponds a map $\varphi^\vee : X \to \{Y \to Z\}$ defined by $(\varphi^\vee(x))\,(y) = \varphi(x, y)$.

Let X, Y and Z be topological spaces.

a. Prove that if $\varphi : X \times Y \to Z$ is contnuous, then $\varphi^\vee(x) : Y \to Z$ is continuous for any $x \in X$.

For topological spaces A, B denote by $\mathcal{C}(A, B)$ the space of continuous maps $X \to Y$ with compact-open topology.

b. Prove that the map $\mathcal{C}(X \times Y, Z) \to \mathcal{C}(X, (Y, Z)) : \varphi \mapsto \varphi^\vee$ is continuous.

c. Prove that if Y is regular and locally compact, then the map $\mathcal{C}(X \times Y, Z) \to \mathcal{C}(X, (Y, Z)) : \varphi \mapsto \varphi^\vee$ is a homeomorphism.

2. Let C be a pointed space such that $X \mapsto [X, C]$ has a natural group structure for any pointed space X (***naturality*** means that for any continuous map $f : X \to Y$ the induced map $f^* : [Y, C] \to [X, C]$ is a homomorphism for these group structures), then C is an H-group and this H-group structure defines the natural group structures in $[X, C]$.