Stony Brook University **Introduction to Advanced Mathematics**
Mathematics Department MAT 250
Oleg Viro Fall 2022

Syllabus

Course description: An introduction to the Advanced Track mathematics program. Provides the core of basic logic, elementary set theory and language of maps. The rigorous language will be applied to define and study some notions of number theory, combinatorics, elementary analysis, Euclidean geometry, topology, etc. No preliminary knowledge of advanced mathematics is required. MAT 250 serves as an alternative to MAT 200 for students in the Advanced Track.

Credits: 4.

Instructor: Oleg Viro. e-mail: oleg.viro@stonybrook.edu
Online office hours: Wednesday at 5pm-6pm, Thursday at 7pm-8pm.

Grader: Matthew Huynh
e-mail: Matthew.Huynh@stonybrook.edu
Office Hours: M 5:00pm-6:00pm zoom
Math Learning Center Hours: (in Math Tower S-235 or online) W,Th 5:00pm-6:00pm zoom

Meetings: TuTh 1:15pm-2:35pm in Physics P117.

Brightspace. All course information (besides homework) will be posted to the Brightspace. Check Announcements regularly!

Textbook: Peter J. Eccles, *An Introduction to Mathematical Reasoning*, Cambridge University Press.

Quizzes will be taken in class. The main goal of quizzes is to make sure that students are aware of definitions and statements of theorems. It will be conducted through Gradescope.

Homework will be assigned weekly through **Gradescope**. The emphasis of the course is on writing proofs, so please try to write legibly and explain your reasoning clearly and fully. You are encouraged to discuss the homework problems with others, but your write-up must be your own work. Suspiciously similar papers won't be graded.

Homework should be submitted to Gradescope according to the Gradescope rules. Incorrect submission format will lead to a grade reduction. Please sign up for Gradescope (https://www.gradescope.com) using **Entry Code** 7GDDN4.

Late homework won't be accepted. Homework in the form of e-mail won't be accepted.

Exams: two midterms and final exam on Monday, Tuesday 12/13, 2:15pm-5:00pm. Missing any of the exams without any serious and documented reason will result to failure in the course.

Grading system: your grade for the course will be based on: homeworks 10%, quizes 10%, class active participation 5%, the first midterm 15%, the second midterm 25%, and the final exam 35%. The final grade is the maximum of the score for final exam and the total grade calculated according to the scheme discribed above.

☞ All your work should be done by you and nobody else. Submitting somebody's else work is a serious violation of university integrity policy and will be treated respectively. See Academic integrity statement below.

Make-up policy: Make-up examinations are given only for work missed due to unforeseen circumstances beyond the student's control.

Student Accessibility Support Center (SASC) statement: If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact SASC (631) 632-6748 or http://studentaffairs.stonybrook.edu/dss/. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and SASC. For procedures and information go to the following website:
http://www.stonybrook.edu/ehs/fire/disabilities/asp.

Academic integrity statement: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at http://www.stonybrook.edu/uaa/academicjudiciary

Critical incident management: Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn.

Student Absences Statement: Students are expected to attend every class, report for examinations and submit major graded coursework as scheduled. If a student is unable to attend lecture(s), report for any exams or complete major graded coursework as scheduled due to extenuating circumstances, the student must contact the instructor as soon as possible. Students may be requested to provide documentation to support their absence and/or may be referred to the Student Support Team for assistance. Students will be provided reasonable accommodations for missed exams, assignments or projects due to significant illness, tragedy or other personal emergencies. In the instance of missed lectures or recitations, the student is responsible for review posted slides, recorded lectures, and notes. Please note, all students must follow Stony Brook, local, state and Centers for Disease Control and Prevention (CDC) guidelines to reduce the risk of transmission of COVID. For questions or more information click here:
https://www.stonybrook.edu/commcms/comingback/students.php

Learning objectives

The goal of the course is to develop student's skills in logical reasoning, proofs and understanding of mathematical language. Students should learn to express themselves in logically correct and mathematically literate written and colloquial language. Students should

- be able to identify propositions and understand logical connectives (negation, conjunction, disjunction, implication, and equivalence) and quantifiers;
- be able to operate with truth tables, analyze and construct propositional forms involving connectives and quantifiers;
- be able to construct logically meaningful sentences using predicates, connectives and quantifiers;
- use correctly logical symbols, formulate and prove logical identities, including tautology and contradiction, de Morgan's laws, the law of excluded middle and the law of consistency;
- recognize logical connectives in colloquial expressions like and, but, though, nevertheless, either ... or;
- recognize conditional and biconditional sentences and use appropriately various colloquial expressions associated with conditionals and biconditionals (sufficient, necessary, sufficient and necessary, whenever, if and only if etc.);
- understand the difference between implication in mathematics and causation in language/everyday life;
- be able to construct the contrapositive, the converse, and the inverse of a conditional statement;
- understand and use correctly quantifiers and use appropriately different colloquial expressions associated with quantifiers;
- be able to translate propositions formulated in plain English into symbolic forms and the other way around;
- be able to analyze and construct sentences involving several quantifiers;
- understand when quantifiers commute and when they don't;
- be able to construct useful denials of propositional forms and quantified sentences;
- understand the logical structure of a definition; in particular, understand mathematical definitions as biconditional sentences;
- understand the logical structure of a mathematical theorem and distinguish the formulation (statement) of a theorem from its proof;
- understand what a counterexample is, understand when and why examples can't replace a proof;
- be able to understand, construct and write proofs of different types: direct proof, proof by contraposition, proof by contradiction, proof by exhaustion;
- recognize and avoid typical logical mistakes of affirming the consequent and denying the antecedent;
- understand, construct and write proofs using principle of mathematical induction in different forms (induction, strong induction, well-ordering principle);
- fluently operate with basic notions of set theory: set and its elements, empty set, subset, intersection, union, difference and complement;

- relate logical and set-theoretical operations, like negation and complement, conjunction and intersection, etc.;
- formulate and prove set-theoretical identities;
- be familiar with the notion of a power set;
- know the definition and properties of the Cartesian product of sets;
- know the definition of a binary relation on a set, understand and use correctly the terminology associated with binary relations: reflexivity, irreflexivity, symmetry, antisymmetry, transitivity;
- know the definitions and examples of strict, non-strict partial order and linear order;
- be familiar with the notions of equivalence relation, partition of a set, equivalence classes, quotient set and the theorem about one-to-one correspondence between equivalence relations on a set and partitions of the set;
- apply the notions of equivalence relation, equivalence classes, quotient set and partition to congruences of integers and modular arithmetics;
- understand and fluently use basic terminology associated with maps: domain, codomain, range, image and preimage of a set, graph of a map;
- know the definition and properties of a composition of maps;
- understand and apply the notions of injection, surjection and bijection;
- know the definition an properties of inverse map, equivalence between invertibility and bijectivity;
- know basic examples of numerical functions and their inverses: exponential and logarithmic, tangent and arctangent, etc;
- understand and use maps of special types: identity, constant, inclusion maps, restriction of a map, submap, maps associated to a Cartesian product, and a quotient set;
- know the definition and properties of characteristic function of a set and a metric on a set;
- know the definitions of equipotent sets and cardinality of a set;
- be able to prove that $\mathbb{N}$ and $\mathbb{Z}$ have the same cardinality;
- be able to prove that any two open segments have the same cardinality;
- know the definitions of arithmetic operations with cardinal numbers and be able to prove the basic properties of these operations;
- know the definition end elementary properties of a finite set;
- be able to formulate, prove and use the pigeon hole principle, its reformulations and corollaries;
- be able to identify natural numbers and cardinalities of finite sets;
- know the definitions of denumerable, countable and uncountable sets;
- formulate and prove basic facts about arithmetics of denumerable sets; understand why $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and their subsets, disjoint unions and direct products are denumerable;
- be able to formulate and prove Cantor's theorem about uncountability of $\mathbb{R}$;
- be able to formulate and prove Cantor's theorem about cardinalities of a set and its power set;
- be able to formulate the continuum hypothesis;
- be able to formulate and prove Cantor-Schröder-Bernstein theorem;
- understand the ordering of cardinal numbers.