MAT 126 Summer II 2009

Practice Final Exam Answers

1. Compute the following indefinite integrals. Don't forget the "+C"!

(a)

$$\int 3x^{-1} - 7\sqrt{x} + x^{2009} + e^{\sqrt{\pi}}dx$$

$$3\ln|x| - \frac{14}{3}x^{\frac{3}{2}} + \frac{x^{2010}}{2010} + e^{\sqrt{\pi}}x + C$$

(b)

$$\int \frac{\ln(y)}{y}dy$$

$$\begin{aligned} u &= \ln(y) \\ du &= \frac{dy}{y} \\ \int \frac{\ln(y)}{y}dy &= \int u du \\ &= \frac{u^2}{2} + C \\ &= \frac{(\ln(y))^2}{2} + C \end{aligned}$$

(c)

$$\int \frac{\ln(y)}{y^2}dy$$

$$u = \ln(y) \qquad dv = \frac{1}{y^2}dy$$

$$du = \frac{1}{y}dy \qquad v = -\frac{1}{y}$$

$$\begin{aligned} \int \frac{\ln(y)}{y^2}dy &= -\frac{\ln(y)}{y} - \int -\frac{1}{y^2} \\ &= -\frac{\ln(y)}{y} - \frac{1}{y} + C \end{aligned}$$

(d)

$$\int te^{-t^2}dt$$

$$\begin{aligned}
u &= -t^2 \\
du &= -2tdt \\
\int te^{-t^2}dt &= -\frac{1}{2}\int e^u du \\
&= -\frac{1}{2}e^u + C \\
&= -\frac{1}{2}e^{-t^2} + C
\end{aligned}$$

(e)

$$\int x^2 \sin(2x)dx$$

$$\begin{aligned}
u = x^2 \qquad & dv = \sin(2x)dx \\
du = 2xdx \qquad & v = -\frac{1}{2}\cos(2x) \\
\int x^2 \sin(2x)dx &= -\frac{x^2}{2}\cos(2x) - \int x\cos(2x)dx \\
u = x \qquad & dv = \cos(2x)dx \\
du = dx \qquad & v = \frac{1}{2}\sin(2x) \\
\int x^2 \sin(2x)dx &= -\frac{x^2}{2}\cos(2x) - \left(\frac{x}{2}\sin(2x) - \int \frac{1}{2}\sin(2x)dx\right) \\
&= -\frac{x^2}{2}\cos(2x) - \left(\frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x)dx\right) + C \\
&= -\frac{x^2}{2}\cos(2x) - \frac{x}{2}\sin(2x) - \frac{1}{4}\cos(2x)dx + C
\end{aligned}$$

(f)

$$\int x^4 \cos(x^2)dx$$

$$\begin{aligned}
w &= x^2 dx \\
dw &= 3x^2 dx \\
\int x^4 \cos(x^2)dx &= \frac{1}{3} w \cos(w) dw \\
u = w & \qquad dv = \cos(w)dw \\
du = dw & \qquad v = \sin(w) \\
\int x^4 \cos(x^2)dx &= \frac{1}{3}\left(w \sin(w) - \int \sin(w)dw\right) \\
&= \frac{1}{3}\left(w \sin(w) + \cos(w)\right) + C \\
&= \frac{1}{3} x^2 \sin(x^2) + \cos(x^2) + C
\end{aligned}$$

(g)

$$\int x \sin(x) e^{\cos(x^2)} dx$$

$$\begin{aligned}
u &= \cos(x^2) \\
du &= -2x \sin(x) dx \\
\int x \sin(x) e^{\cos(x^2)} dx &= -\frac{1}{2} e^u du \\
&= -\frac{1}{2} e^u + C \\
&= -\frac{1}{2} e^{\cos(x^2)} + C
\end{aligned}$$

(h)

$$\int \frac{x^2}{\sqrt{4-x^3}}dx$$

$$\begin{aligned} u &= 4-x^3 \\ du &= -3x^2dx \\ \int \frac{x^2}{\sqrt{4-x^3}}dx &= -\frac{1}{3}\int \frac{du}{\sqrt{u}} \\ &= -\frac{1}{3}\int u^{-\frac{1}{2}}du \\ &= -\frac{2}{3}u^{\frac{1}{2}}+C \\ &= -\frac{2}{3}\sqrt{4-x^3}+C \end{aligned}$$

(i)

$$\int e^x \sin\left(\frac{x}{2}\right)dx$$

$$\begin{aligned} u = e^x & \qquad dv = \sin\left(\frac{x}{2}\right)dx \\ du = e^x dx & \qquad v = -2\cos\left(\frac{x}{2}\right) \\ \int e^x \sin\left(\frac{x}{2}\right)dx &= -2e^x\cos\left(\frac{x}{2}\right) - \int -2e^x\cos\left(\frac{x}{2}\right)dx + C \\ u = e^x & \qquad dv = \cos\left(\frac{x}{2}\right)dx \\ du = e^x dx & \qquad v = 2\sin\left(\frac{x}{2}\right) \\ \int e^x \sin\left(\frac{x}{2}\right)dx &= -2e^x\cos\left(\frac{x}{2}\right) + 2\left(2e^x\sin\left(\frac{x}{2}\right) - \int 2e^x\sin\left(\frac{x}{2}\right)dx\right) + C \\ \int e^x \sin\left(\frac{x}{2}\right)dx &= -2e^x\cos\left(\frac{x}{2}\right) + 4e^x\sin\left(\frac{x}{2}\right) - 4\int e^x\sin\left(\frac{x}{2}\right)dx + C \\ 5\int e^x \sin\left(\frac{x}{2}\right)dx &= -2e^x\cos\left(\frac{x}{2}\right) + 4e^x\sin\left(\frac{x}{2}\right) + C \\ \int e^x \sin\left(\frac{x}{2}\right)dx &= -\frac{2}{5}e^x\cos\left(\frac{x}{2}\right) + \frac{4}{5}e^x\sin\left(\frac{x}{2}\right) + C \end{aligned}$$

(j)

$$\int \tan^{-1}(x)dx$$

$$u = \tan^{-1}(x) \qquad dv = dx$$

$$du = \frac{dx}{1+x^2} \qquad v = x$$

$$\begin{aligned}
\int \tan^{-1}(x)dx &= x\tan^{-1}(x) - \int \frac{x}{1+x^2}dx \\
u &= 1+x^2 \\
du &= 2xdx \\
\int \tan^{-1}(x)dx &= x\tan^{-1}(x) - \int \frac{1}{2u}du \\
&= x\tan^{-1}(x) - \frac{1}{2}\ln|u| + C \\
&= x\tan^{-1}(x) - \ln\sqrt{1+x^2} + C
\end{aligned}$$

(k)

$$\int \frac{w}{2-w}dw$$

$$\begin{aligned}
u &= 2-w \\
du &= -dw \\
\int \frac{w}{2-w}dw &= -\int \frac{2-u}{u}du \\
&= -\int \frac{2}{u} - \frac{u}{u}du \\
&= -\ln|u| + u + C \\
&= -\ln|2-w| + 2 - w + C
\end{aligned}$$

(l)

$$\int \frac{e^{-2z}-1}{e^z}dz$$

$$\begin{aligned}
\int \frac{e^{-2z}-1}{e^z}dz &= \int \frac{e^{-2z}}{e^z} - \frac{1}{e^z}dz \\
&= \int e^{-3z} - e^{-z}dz \\
&= -\frac{1}{3}e^{-3z} + e^{-z} + C
\end{aligned}$$

(m)

$$\int \sin^3(y)dy$$

$$\begin{aligned}
\int \sin^3(y)dy &= \int \sin(y)\left(1-\cos^2(y)\right)dy \\
u &= \cos(y) \\
du &= -\sin(y)dy \\
\int \sin^3(y)dy &= -\int (1-u^2)du \\
&= -u + \frac{u^3}{3} + C \\
&= -\cos(y) + \frac{\cos^3(y)}{3} + C
\end{aligned}$$

(n)

$$\int \sqrt{9-x^2}dx$$

$$\begin{aligned}
x &= 3\sin\theta \\
dx &= 3\cos\theta d\theta \\
\int \sqrt{9-x^2}dx &= \int \sqrt{9-(3\sin\theta)^2}3\cos\theta d\theta \\
&= 3\int \sqrt{1-\sin^2\theta}3\cos\theta d\theta \\
&= 9\int \cos^2\theta d\theta \\
&= 9\int \frac{1+\cos(2\theta)}{2}d\theta \\
&= \frac{9}{2}\theta + \frac{9}{4}\sin(2\theta) + C \\
&= \frac{9}{2}\theta + \frac{18}{4}\sin(\theta)\cos(\theta) + C \\
&= \frac{9}{2}\sin^{-1}(x) + \frac{18}{4}\frac{x}{3}\frac{\sqrt{9-x^2}}{3} + C \\
&= \frac{9}{2}\sin^{-1}(x) + \frac{1}{2}x\sqrt{9-x^2} + C
\end{aligned}$$

(o)

$$\int \frac{5x-1}{x^2+2x-35}dx$$

$$\begin{aligned}
\frac{5x-1}{x^2+2x-35} &= \frac{5x-1}{(x+7)(x-5)} \\
\frac{5x-1}{(x+7)(x-5)} &= \frac{A}{x+7}+\frac{B}{x-5} \\
5x-1 &= A(x-5)+B(x+7) \\
5x-1 &= Ax-5A+Bx+7B \\
5x-1 &= (A+B)x-5A+7B \\
& \left\{ \begin{array}{rcr} 5 &=& A+B \\ -1 &=& -5A+7B \end{array} \right. \\
& \left\{ \begin{array}{rcl} B &=& 2 \\ A &=& 3 \end{array} \right. \\
\int \frac{5x-1}{x^2+2x-35}dx &= \int \frac{3}{x+7}+\frac{2}{x-5}dx \\
&= 3\ln|x+7|+2\ln|x-5|+C
\end{aligned}$$

2. Compute the following definite integrals. Some may diverge; in this case show why and write "divergent".

(a)

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(2x)dx$$

$$\begin{aligned}
\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(2x)dx &= -\frac{1}{2}\cos(2x)\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \\
&= -\frac{1}{2}\left(\cos(2(\frac{\pi}{2})) - \cos(2(-\frac{\pi}{3}))\right) \\
&= -\frac{1}{2}\left(-1 - (-\frac{1}{2})\right) \\
&= \frac{1}{4}
\end{aligned}$$

(b)

$$\int_{2}^{\infty} \frac{1}{x^3}dx$$

$$\begin{aligned}
\int_{2}^{\infty} \frac{1}{x^3}dx &= \lim_{t\to\infty} \int_{2}^{t} \frac{1}{x^3}dx \\
&= \lim_{t\to\infty} \frac{1}{-2x^2}\Big|_{2}^{t} \\
&= \lim_{t\to\infty} \frac{1}{-2t^2} - \frac{1}{-2(2^2)} \\
&= 0 + \frac{1}{8} \\
&= \frac{1}{8}
\end{aligned}$$

(c)

$$\int_{0}^{2} \frac{1}{x^3}dx$$

$$\begin{aligned}
\int_{0}^{2} \frac{1}{x^3}dx &= \lim_{t\to 0^+} \int_{0}^{2} \frac{1}{x^3}dx \\
&= \lim_{t\to 0^+} \frac{1}{-2x^2}\Big|_{t}^{2} \\
&= \lim_{t\to 0^+} \frac{1}{-2(2^2)} - \frac{1}{-2(t^2)} \\
&= \frac{1}{8} + \infty
\end{aligned}$$

DIVERGES

(d)

$$\int_{-\infty}^{\infty} xe^{-4x^2}dx$$

$$\begin{aligned}
\int_{-\infty}^{\infty} xe^{-4x^2}dx &= \lim_{a\to-\infty}\int_a^0 xe^{-4x^2}dx + \lim_{b\to\infty}\int_0^b xe^{-4x^2}dx \\
u &= -4x^2 \\
du &= -8xdx \\
\int_{-\infty}^{\infty} xe^{-4x^2}dx &= \lim_{a\to-\infty}\int_a^0 -\frac{1}{8}e^u du + \lim_{b\to\infty}\int_0^b -\frac{1}{8}e^u du \\
&= \lim_{a\to-\infty} -\frac{1}{8}e^{-4x^2}\Big|_a^0 + \lim_{b\to\infty}\Big|_0^b -\frac{1}{8}e^{-4x^2} \\
&= \lim_{a\to-\infty} -\frac{1}{8}(e^{-4(0)^2} - e^{-4(a)^2}) + \lim_{b\to\infty} -\frac{1}{8}(e^{-4b^2} - e^{-4(0)^2}) \\
&= -\frac{1}{8} + \frac{1}{8} \\
&= 0
\end{aligned}$$

(e)

$$\int_0^3 \frac{2}{\sqrt[3]{1-x}}dx$$

$$\begin{aligned}
\int_0^3 \frac{2}{\sqrt[3]{1-x}}dx &= \lim_{t\to1^-}\int_0^t \frac{2}{\sqrt[3]{1-x}}dx + \lim_{t\to1^+}\int_t^3 \frac{2}{\sqrt[3]{1-x}}dx \\
&= \lim_{t\to1^-} 3(1-x)^{\frac{2}{3}}\Big|_0^t + \lim_{t\to1^+} 3(1-x)^{\frac{2}{3}}\Big|_t^3 \\
&= \lim_{t\to1^-} 3(1-t)^{\frac{2}{3}} - 3(1-0)^{\frac{2}{3}} + \lim_{t\to1^+} 3(1-3)^{\frac{2}{3}} - 3(1-t)^{\frac{2}{3}} \\
&= -3 + 3(-2)^{\frac{2}{3}}
\end{aligned}$$

3. Determine if each of the following integrals converges or diverges. Justify your answer.

(a)

$$\int_0^\pi \frac{1}{x-\sin(x)}dx$$

For $0 \le x \le \pi$:

$$\begin{aligned} \sin(x) &\ge 0 \\ x-\sin(x) &\le x \\ \frac{1}{x-\sin(x)} &\ge \frac{1}{x} \end{aligned}$$

We compute:

$$\begin{aligned} \int_0^\pi \frac{1}{x}dx &= \lim_{t\to 0^+}\int_t^\pi \frac{1}{x} \\ &= \lim_{t\to 0^+} \ln|x|\Big|_t^\pi \\ &= \lim_{t\to 0^+} \ln|\pi| - \ln|t| \\ &= \ln|\pi| + \infty \end{aligned}$$

We have shown $\int_0^\pi \frac{1}{x}dx$ diverges, so $\int_0^\pi \frac{1}{x-\sin(x)}dx$ divereges as well.

(b)

$$\int_1^\infty \frac{x}{\sqrt{x^8+7x^2+2}}dx$$

For $x \ge 1$:

$$\begin{aligned} x^8+7x^2+2 &\ge x^8 \\ \frac{1}{\sqrt{x^8+7x^2+2}} &\le \frac{1}{\sqrt{x^8}} \\ \frac{x}{\sqrt{x^8+7x^2+2}} &\le \frac{x}{\sqrt{x^8}} \\ &= \frac{x}{x^4} \\ &= \frac{1}{x^3} \end{aligned}$$

We know $\int_1^\infty \frac{1}{x^3}dx$ converges, so $\int_1^\infty \frac{x}{\sqrt{x^8+7x^2+2}}dx$ converges as well.

(c)

$$\int_{-\infty}^{\infty} e^{x^2} dx$$

First observe

$$\int_{-\infty}^{\infty} e^{x^2} dx = \int_{-\infty}^{0} e^{x^2} dx + \int_{0}^{\infty} e^{x^2} dx$$

For $x < 0$, $e^{-x} \leq e^{x^2}$. We compute:

$$\begin{aligned}
\int_{-\infty}^{0} e^{-x} dx &= \lim_{t \to -\infty} \int_{t}^{0} e^{-x} dx \\
&= \lim_{t \to -\infty} -e^{-x} \Big|_{t}^{0} \\
&= \lim_{t \to -\infty} -e^{-t} + e^{-0} \\
&= -\infty
\end{aligned}$$

We have shown that $\int_{-\infty}^{0} e^{-x} dx$ diverges, so $\int_{-\infty}^{0} e^{x^2} dx$ diverges and thus $\int_{-\infty}^{\infty} e^{x^2} dx$ must also diverge.

4. Consider the definite integral $\int_0^4 x^2 dx$.

(a) Compute L_4, the left handed approximation of $\int_0^4 x^2 dx$ using 4 subintervals.

$$\begin{aligned} L_4 &= 1\left(0^2 + 1^2 + 2^2 + 3^2\right) \\ &= 14 \end{aligned}$$

(b) Compute T_4, the trapezoidal approximation of $\int_0^4 x^2 dx$ using 4 subintervals.

$$\begin{aligned} T_4 &= \frac{1}{2}\left(0^2 + 2 \cdot 1^2 + 2 \cdot 2^2 + 2 \cdot 3^2 + 4^2\right) \\ &= 22 \end{aligned}$$

(c) Compute S_4, the approximation of $\int_0^4 x^2 dx$ using Simpson's Rule with 4 subintervals.

$$\begin{aligned} S_4 &= \frac{1}{3}\left(0^2 + 4 \cdot 1^2 + 2 \cdot 2^2 + 4 \cdot 3^2 + 4^2\right) \\ &= \frac{64}{3} \end{aligned}$$

(d) Use the error formula

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

to give the best upper bound you can on the error in the approximation T_4. If $f(x) = x^2$, $f''(x) = 2$. Thus the error is bounded above by

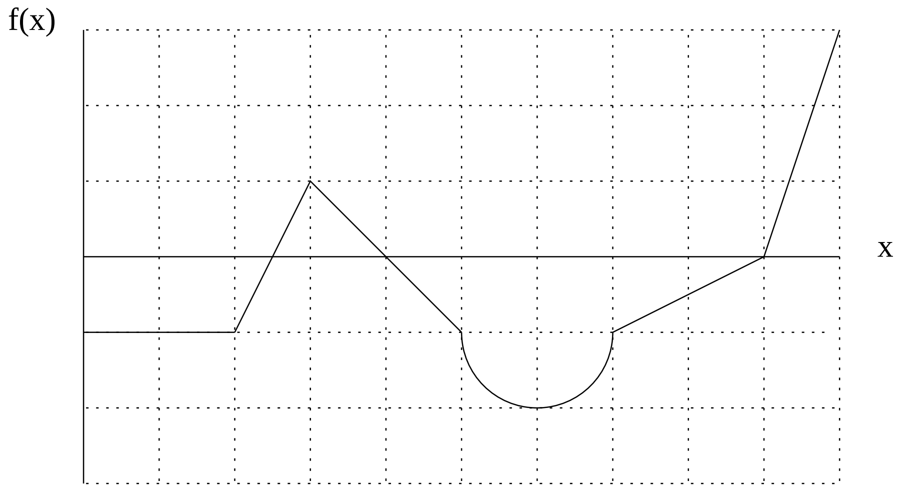
$$\frac{2(4-0)^3}{12(4)^2} = .75$$

(e) How large must n for M_n to approximate $\int_0^4 x^2 dx$ to within .001?

$$\begin{aligned} \frac{2(4-0)^3}{24(n)^2} &\leq .001 \\ \frac{128}{24(.001)} &\leq n^2 \\ \sqrt{\frac{128}{24(.001)}} &\leq n \end{aligned}$$

5. Below is the graph of a function $f(x)$, continuous on the interval $[0, 10]$. Define a new function $g(x)$ by

$$g(x) = \int_0^x f(t)dt$$



(a) On what intervals is $g(x)$ increasing?
(2.5,4) and (9,10)

(b) Compute $g(7)$. $-4 - \frac{\pi}{2}$

(c) Compute $g'(3)$. 1

(d) List the x-values of any local minima of $g(x)$, or write "NONE" if there are none. $x = 2.5, x = 9$

(e) On what intervals is $g(x)$ concave down? $(3, 6)$

(f) List the x-values of any inflection points of $g(x)$, or write "NONE" if there are none. $x = 4, x = 6$

(g) What is the average value of $f(x)$ on the interval $[0, 10]$? $\frac{-7-\pi}{20}$

6. (a) Find the area of the region enclosed by the curves $y = x^2$ and $y = x^3$.

$$\begin{aligned} A &= \int_0^1 x^2 - x^3 dx \\ &= \frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 \\ &= \frac{1^3}{3} - \frac{1^4}{4} \\ &= \frac{1}{12} \end{aligned}$$

(b) Find the volume of the solid created when the above region is rotated around the line $x = -1$.

$$\begin{aligned} V &= 2\pi \int_0^1 (1+x)(x^2 - x^3) dx \\ &= 2\pi \int_0^1 x^2 - x^3 + x^3 - x^4 dx \\ &= 2\pi \left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 \\ &= 2\pi \left(\frac{1^3}{3} - \frac{1^5}{5}\right) \\ &= \frac{4\pi}{15} \end{aligned}$$

(c) Find the volume of the solid created when the above region is rotated around the line $y = -2$.

$$\begin{aligned} V &= \pi \int_0^1 (2+x^2)^2 - (2+x^3)^2 dx \\ &= \pi \int_0^1 4 + 4x^2 + x^4 - 4 - 4x^3 - x^6 dx \\ &= \pi \left(\frac{4}{3}x^3 + \frac{x^5}{5} - x^4 - \frac{x^7}{7}\right)\Big|_0^1 \\ &= \pi \left(\frac{4}{3} + \frac{1}{5} - 1 - \frac{1}{7}\right) \end{aligned}$$

(d) Find the volume of the solid whose base is the above region and with cross sections perpendicular to the x-axis squares.

$$\begin{aligned} V &= \int_0^1 (x^2 - x^3)^2 dx \\ &= \int_0^1 x^4 - 2x^5 + x^6 \\ &= \left(\frac{x^5}{5} - 2\frac{x^6}{6} + \frac{x^7}{7}\right)\Big|_0^1 \\ &= \left(\frac{1}{5} - \frac{2}{6} + \frac{1}{7}\right) \end{aligned}$$

7. Define a function $h(x)$ by:

$$h(x) = \int_{4x}^{\sin(2x)} \frac{\cos(t)}{t} dt$$

Find $h'(x)$.

$$\frac{\cos(\sin(2x))}{\sin(2x)}(2\cos(2x)) - \frac{\cos(4x)}{4x} \cdot 4$$

8. Write down a definite integral which computes the length of the curve $y = \sin(x)$ between $x = 0$ and $x = \pi$.

$$\int_0^{\pi} \sqrt{1 + \cos^2(x)}dx$$

9. Suppose that $\rho(x)$ represents the density of a thin rod in kg/cm, where x is measured in cm. What does

$$\int_5^{10} \rho(x)dx$$

represent? Include units.
$\int_5^{10} \rho(x)dx$ represents the mass of the rod in kg between $x = 5$ and $x = 10$.

10. A function $f(x)$ is given by:

$$f(x) = \begin{cases} 0 & x < 1 \\ \frac{k}{x^4} & x \geq 1 \end{cases}$$

(a) For what value of k is $f(x)$ a probability density function?

$$\begin{aligned} \int_{-\infty}^{\infty} f(x)dx &= \int_{1}^{\infty} \frac{k}{x^4}dx \\ &= \lim_{t\to\infty} \int_{1}^{t} \frac{k}{x^4}dx \\ &= \lim_{t\to\infty} -\frac{k}{3x^3}\Big|_1^t \\ &= \lim_{t\to\infty} -\frac{k}{3t^3} + \frac{k}{3(1)^3} \\ &= 0 + \frac{k}{3} \end{aligned}$$

The only value of k for whic this integral is equal to 1 is $k = 3$.

(b) What is the mean of the random variable X associated to the probability density function $f(x)$?

$$\begin{aligned} \int_{-\infty}^{\infty} xf(x)dx &= \int_{1}^{\infty} \frac{3x}{x^4}dx \\ &= \lim_{t\to\infty} \int_{1}^{t} \frac{3x}{x^4}dx \\ &= \lim_{t\to\infty} \int_{1}^{t} \frac{3}{x^3}dx \\ &= \lim_{t\to\infty} -\frac{3}{2x^2}\Big|_1^t \\ &= \lim_{t\to\infty} -\frac{3}{2t^2} + \frac{3}{2(1)^3} \\ &= 0 + \frac{3}{2} \end{aligned}$$

(c) What is the median of the random variable X associated to the probability density function $f(x)$?

$$\begin{aligned}
\frac{1}{2} &= \int_{-\infty}^{m} f(x)dx \\
\frac{1}{2} &= \int_{1}^{m} \frac{3}{x^4}dx \\
\frac{1}{2} &= -\frac{1}{x^3}\Big|_1^m \\
\frac{1}{2} &= -\frac{1}{m^3} + \frac{1}{1^3} \\
\frac{1}{2} &= \frac{1}{m^3} \\
m &= \sqrt[3]{2}
\end{aligned}$$

11. A hemispherical tank is 10 meters deep, and is full of water. (The tank looks like the bottom half of a sphere.) How much work does it take to pump all of the water out of the top of tank? (Water has a density of 1000 $\frac{kg}{m^3}$.)

Put the orgin at the center of the top of the tank. Then a circlular slice of water x meters deep and dx meters thick has volume $\pi r^2 dx$ cubic meters. Similar triangles shows that $r = 10 - x$. Thus the volume of the slice is

$$\pi(10 - x)^2 dx \;\; m^3$$

which has mass

$$\left(1000\frac{kg}{m^3}\right)\left(\pi(10 - x)^2 dx \;\; m^3\right)$$

which exherts a force of:

$$\left(9.8\frac{m}{s^2}\right)\left(1000\frac{kg}{m^3}\right)\left(\pi(10 - x)^2 dx \;\; m^3\right)$$

The work required to lift this slice to the top of the tank is then:

$$9800\pi x(10 - x)^2 dx \;\; J$$

The total work is:

$$\begin{aligned} 9800\pi \int_0^{10} x(10 - x)^2 dx &= 9800\pi \int_0^{10} 100x - 20x^2 + x^3 dx \\ &= 50x^2 - \frac{20x^3}{3} + \frac{x^4}{4}\Big|_0^1 0 \\ &= 50(100) - 20000/3 + 10000/4 \;\; N \\ &= 833.\bar{3} \;\; N \end{aligned}$$

12. A car has initial velocity 10 ft/s and accelerating at a rate of 12 ft/s^2. How far does the car travel in 3 seconds?

$$\begin{aligned} a(t) &= 12 \\ v(t) &= 12t + 10 \\ x(t) &= 6t^2 + 10t \\ x(3) &= 6(3)^2 + 10(3) \\ &= 84 \end{aligned}$$