1. Below are some statements about functions. If the statement is *always true* no matter what the function is, cicle "TRUE". If the statement is *sometimes true and sometimes false*, depending on what the function is, circle "FALSE".

(a) If $f(x)$ is continuous on the open interval (a, b), then $f(x)$ has an abolute maximum on the interval (a, b)

FALSE

(b) If $g(x)$ has a local maximum at $x = 3$ then $g'(3) = 0$

FALSE

(c) If $g'(3) = 0$, then $g(x)$ has a local maximum at $x = 3$

FALSE

(d) If $h(x)$ is differentiable on the interval $[1, 3]$, $h(1) = 3$ and $h(2) = 7$, then for some number c between 1 and 3, $h'(c) = 2$

TRUE

(e) If $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = 17$, then $f(x)$ is continuous at $x = 3$

TRUE

(f) If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$

FALSE

(g) If $g(x)$ is a 1-to-1 dfferentiable function, then $g^{-1}(x)$ exists and is differentiable

FALSE

(h) If $f(x)$ is a continuous function and a and b are real numbers in the domain of $f(x)$, then $f(a+b) = f(a) + f(b)$

FALSE

(i) If $f(x)$ is a continuous function, $f(1) = 4$, and $f(3) = 12$, then $4 \leq f(2) \leq 12$

FALSE

(j) The graph of a continuous function can cross a vertical assymtope of the function

FALSE

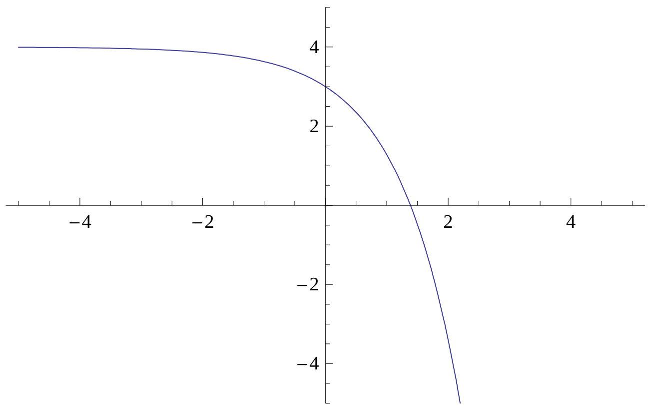
(k) The graph of a continuous function can cross a horizontal assymtope of the function

TRUE

(l) If $\lim_{x\to 7} f(x)g(x) = 24$, then $\lim_{x\to 7} f(x)$ exists

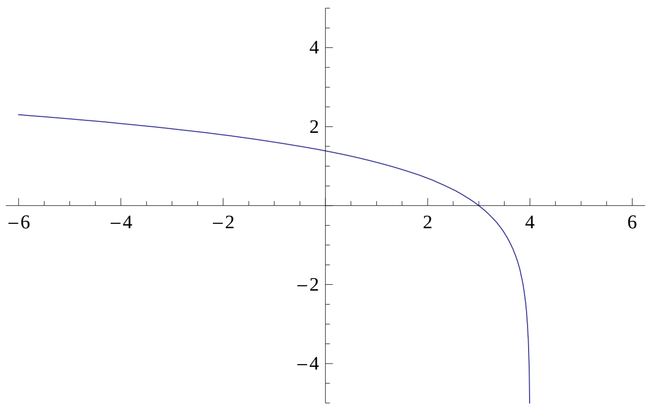
FALSE

2. Consider the function $f(x) = 4 - e^x$
(a) Sketch $f(x)$ on the axes provided, and state the domain and range of $f(x)$. Clearly indicate horitontal and vertical assymtopes on your graph.



Domain = $(-\infty, \infty)$
Range = $(-\infty, 4)$
Horizontal assymtope at $y = 4$

(b) Write a formula for the inverse function of $f(x)$. Then sketch a graph of $f^{-1}(x)$ and state its domain and range.



$f^{-1}(x) = \ln(4 - x)$
Domain = $(-\infty, 4)$
Range = $(-\infty, \infty)$
Vertical assymtope at $x = 4$

3. Compute the following limits. If the limit exists and is a real number, write the number. If a limit does not exist, write "$+\infty$", "$-\infty$", or "DNE" as appropriate. Do not write "undefined", because I won't know what you mean.

(a) $\lim_{x \to \frac{\pi}{2}} e^{\cos x} = e$

(b) $\lim_{x \to 2} \frac{x-3}{x^2-9} = \frac{1}{5}$

(c) $\lim_{x \to 3} \frac{x-3}{x^2-9} = \frac{1}{6}$

(d) $\lim_{x \to \infty} \frac{x-3}{x^2-9} = 0$

(e) $\lim_{t \to 7} \frac{t-2}{t^2-8t+7} \quad DNE$

(f) $\lim_{x\to\infty} \sin(x)$ $\quad DNE$

(g) $\lim_{h\to 0} \frac{\sqrt{h+4}-2}{h} = \frac{1}{4}$

(h) $\lim_{y\to\infty} \sqrt{y^2+7y+2} - y = \frac{7}{2}$
This one is tricky: Multiply by conjugate and then look at ratio of coefficients

(i) $\lim_{u\to\infty} -\ln(u) = -\infty$

(j) $\lim_{t\to\frac{\pi}{2}^+} e^{\tan(t)} = 0$

The limits on this page are trickier. Try the Squeeze Theorem and l'Hospital's rule.

(k) $\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$
l'Hospital

(l) $\lim_{x\to 0} x^2 \sin(x) = 0$
Squeeze

(m) $\lim_{x\to 0^+} \ln(x)\sin(2x) = 0$
Tricky: l'Hosptial, some algebra, and l'Hospital again

(n) $\lim_{x\to\infty} e^{-x}\sin(x) = 0$
Squeeze

(o) $\lim_{x\to 6^+} \frac{2x-12}{|x-6|} = 2$
For $x > 6$, $|x-6| = x-6$. For $x < 6$, $|x-6| = -(x-6)$, so $\lim_{x\to 6^-} \frac{2x-12}{|x-6|} = -2$

4. Differentiate the following funtions with respect to x.

(a) $(7x^7 - 6x^6)^2$
$2(7x^7 - 6x^6)(49x^6 - 36x^5)$

(b) $2\cos(2x)$
$-4\sin(2x)$

(c) xe^{-3x}
$-3xe^{-3x} + e^{-3x}$

(d) $\frac{\ln(x)}{11x^3}$
$\frac{11x^3 \frac{1}{x} - 33x^2 \ln(x)}{(11x^3)^2}$

(e) $\sin(\sqrt{1+e^x})$
$\cos(\sqrt{1+e^x}) \frac{1}{2\sqrt{1+e^x}} e^x$

(f) $\sec(x^2)$
$2x\sec(x^2)\tan(x^2)$

(g) e^3
0
e^3 is a constant!

(h) $x^4 + 4^x + \sin^{-1}(4x)$
$4x^3 + 4^x\ln(4) + \frac{4}{\sqrt{1-(4x)^2}}$

(i) $\log_2(\sqrt[3]{x})$
$\frac{1}{\sqrt[3]{x}\ln(2)}\frac{1}{3}x^{-\frac{2}{3}}$

(j) $\frac{3x-xe^{2x}}{\tan(x)}$
$\frac{\tan(x)(3-2xe^{2x}-e^{2x})-(3x-e^{2x})\sec^2(x)}{\tan^2(x)}$

5. Find $\frac{dy}{dx}$ in each of the following problems:

(a) $y = e^{-x\sin(x^2)}\cos(\sqrt{7x^3 - 3x})$

$$e^{-x\sin(x^2)}(-\sin(\sqrt{7x^3-3x})\frac{1}{2\sqrt{7x^3-3x}}(21x^2-3))$$
$$+(-2x^3\cos(x^2) - \sin(x^2))e^{-x\sin(x^2)}\cos(\sqrt{7x^3-3x})$$

(b) $xy^3 - yx^3 = 2$

$$\frac{dy}{dx} = \frac{3x^3y - y^3}{xy^2 - x^3}$$

(c) $ye^{x+y} = 10x$

$$\frac{dy}{dx} = \frac{10 - ye^{x+y}}{ye^{x+y} + e^{x+y}}$$

6. $f(x)$ and $g(x)$ are differentiable functions such that $f(2) = 3$, $f'(2) = 5$, $g(2) = -2$, $g'(2) = 4$, $f'(-2) = 4$, and $f'(4) = -1$. Compute the following derivatives:

(a) $h(x) = f(x)g(x) \quad h'(2) = f(2)g'(2) + f'(2)g(2) = 2$

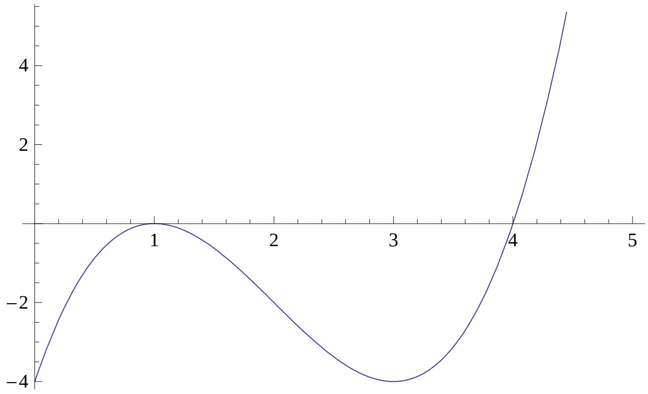
(b) $h(x) = f(g(x)) \quad h'(2) = f'(g(2))g'(2) = -4$

(c) $h(x) = \ln(f(x)) \quad h'(2) = \frac{f'(2)}{f(2)} = \frac{5}{3}$

(d) $h(x) = \frac{x^2}{g(x)} \quad h'(2) = \frac{g(2)2(2)-(2)^2g'(2)}{[g(2)]^2} = -6$

(e) $h(x) = 3[g(x)]^2 + 7[f(x^2)] \quad h'(2) = 6g(2)g'(2) + 7f'((2)^2)(2(2)) = -76$

7. $f(x)$ is a differentiable function defined on the interval $[0, 4.5]$. Here is a graph of $f'(x)$, the **derivative** of $f(x)$. Answer the following questions about $f(x)$.



(a) On what intervals is $f(x)$ increasing? $(4, 4.5)$

(b) On what intervals is $f(x)$ decreasing? $(0, 4)$

(c) List the x-values of any local maxima of $f(x)$, or write "NONE" if there are none. $x = 0, 4.5$
I would also except "NONE" as an answer, as both of the local maxima occur at endpoints of the domain

(d) List the x-values of any local minima of $f(x)$, or write "NONE" if there are none. $x = 4$

(e) On what intervals is $f(x)$ concave up? $(0, 1)$ $(3, 4.5)$

(f) On what intervals is $f(x)$ concave down? $(1, 3)$

(g) List the x-values of any inflection points of $f(x)$, or write "NONE" if there are none. $x = 1, 3$

8. Consider $g(x) = \frac{x^4}{4} - x^3 + x^2$

(a) On what intervals is $g(x)$ increasing? $(0, 1)$ $(2, \infty)$

(b) List the local minima of $g(x)$, or write "NONE" if there are none
$x = 0, 2$

(c) Find the absolute maximum and minimum values of $g(x)$ on the interval $[0, 2]$
$f(0) = 0,\ f(1) = \frac{1}{4}\ f(2) = 0$
Min value is 0 and max value is $\frac{1}{4}$

(d) List the inflection points of $g(x)$
$x = 1 + \frac{\sqrt{2}}{3}, 1 - \frac{\sqrt{2}}{3}$

(e) Where is $g(x)$ concave up?
$(-\infty, 1 - \frac{\sqrt{2}}{3})$ $(1 + \frac{\sqrt{2}}{3}, \infty)$

9. Let $f(x) = \sqrt{x}$

(a) Find the equation of the tangent line to $f(x)$ at $x = 4$

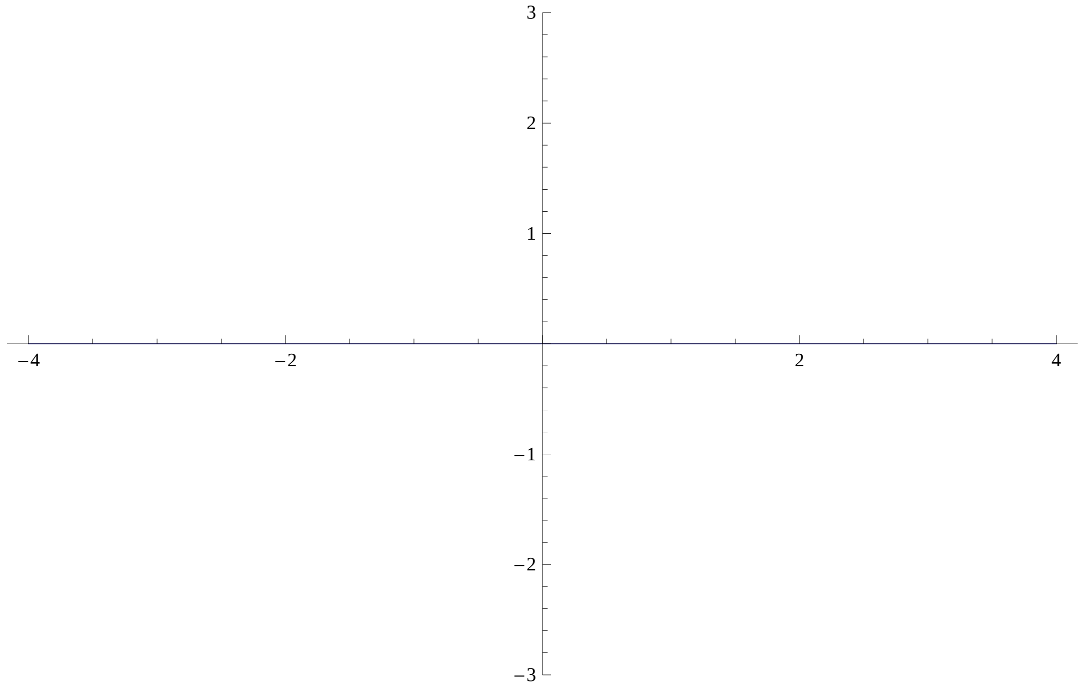
$$L(x) = 2 + \frac{1}{4}(x - 4)$$

(b) Use your from part (a) to approximate $\sqrt{3.9}$

$$L(3.9) = 2 + .25(-.1) = 1.975$$

10. Sketch the graph of a function $f(x)$ satisfying the properties listed below. Make sure your graph is the graph of a function!

- $f(0) = 0$
- $f'(x) > 0$ on $(-1, 1)$
- $f'(x) < 0$ on $(-\infty, -1)$ and $(1, \infty)$
- $f'(1) = f'(-1) = 0$
- $f''(x) > 0$ on $(-2, 0)$ and $(2, \infty)$
- $f''(x) < 0$ on $(-\infty, -2)$ and $(0, 2)$
- $f''(2) = f''(-2) = 0$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 0$



11. Oil oozes out of the bottom of truck into a 2 inch deep cylindrical puddle at a rate of 5 cubic inches per minute. At what rate is the radius of the puddle increasing when the radius is 12 inches? (Recall that the volume of a cycliner of radius r and height h is given by the formula $V = \pi r^2 h$.)

$$\begin{aligned} V &= \pi r^2 h \\ V &= \pi r^2 (2) \\ \frac{dV}{dt} &= 4\pi r \frac{dr}{dt} \\ 5 &= 4\pi(12)\frac{dr}{dt} \\ \frac{5}{48\pi} &= \frac{dr}{dt} \end{aligned}$$

$\frac{5}{48\pi}$ ft/min

12. Find the area of the largest rectangle with two vertices on the x-axis and two vertices on the graph of the parabola $y = 9 - x^2$.

Area $= 2xy$ and the constraint tells us $y = 9 - x^2$.

$$\begin{aligned}
A(x) &= 2x(9 - x^2) \\
&= 18x - 2x^3 \\
A'(x) &= 18 - 6x^2 \\
0 &= 18 - 6x^2 \\
x^2 &= 3 \\
x &= \sqrt{3} \\
A(3) &= 2(\sqrt{3})(9 - (\sqrt{3})^2) \\
&= 12\sqrt{3}
\end{aligned}$$

Maximum area is $12\sqrt{3}$