MAT 126 Summer II 2009

Practice Final Exam

There are 12 questions on this practice exam. This practice exam is **longer** than the real exam; the real exam will have around 8 questions, with fewer parts per question.
Solutions may appear on the course webpage.

1. Compute the following indefinite integrals. Don't forget the "+C"!

(a)

$$\int 3x^{-1} - 7\sqrt{x} + x^{2009} + e^{\sqrt{\pi}} dx$$

(b)

$$\int \frac{\ln(y)}{y} dy$$

(c)

$$\int \frac{\ln(y)}{y^2} dy$$

(d)

$$\int te^{-t^2} dt$$

(e)

$$\int x^2 \sin(2x) dx$$

(f)

$$\int x^4 \cos(x^2) dx$$

(g)

$$\int x \sin(x) e^{\cos(x^2)} dx$$

(h)

$$\int \frac{x^2}{\sqrt{4-x^3}} dx$$

(i)

$$\int e^x \sin\left(\frac{x}{2}\right) dx$$

(j)

$$\int \tan^{-1}(x) dx$$

(k)

$$\int \frac{w}{2-w} dw$$

(l)

$$\int \frac{e^{-2z}-1}{e^z} dz$$

(m)

$$\int \sin^3(y) dy$$

(n)

$$\int \sqrt{9-x^2} dx$$

(o)

$$\int \frac{5x-1}{x^2+2x-35} dx$$

2. Compute the following definite integrals. Some may diverge; in this case show why and write "divergent".

(a)

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(2x)dx$$

(b)

$$\int_{2}^{\infty} \frac{1}{x^3}dx$$

(c)

$$\int_{0}^{2} \frac{1}{x^3}dx$$

(d)

$$\int_{-\infty}^{\infty} xe^{-4x^2}dx$$

(e)

$$\int_{0}^{3} \frac{2}{\sqrt[3]{1-x}}dx$$

3. Determine if each of the following integrals converges or diverges. Justify your answer.

(a)

$$\int_0^{\pi} \frac{1}{x - \sin(x)} dx$$

(b)

$$\int_1^{\infty} \frac{x}{\sqrt{x^8 + 7x^2 + 2}} dx$$

(c)

$$\int_{-\infty}^{\infty} e^{x^2} dx$$

4. Consider the definite integral $\int_0^4 x^2 dx$.

(a) Compute L_4, the left handed approximation of $\int_0^4 x^2 dx$ using 4 subintervals.

(b) Compute T_4, the trapezoidal approximation of $\int_0^4 x^2 dx$ using 4 subintervals.

(c) Compute S_4, the approximation of $\int_0^4 x^2 dx$ using Simpson's Rule with 4 subintervals.

(d) Use the error formula

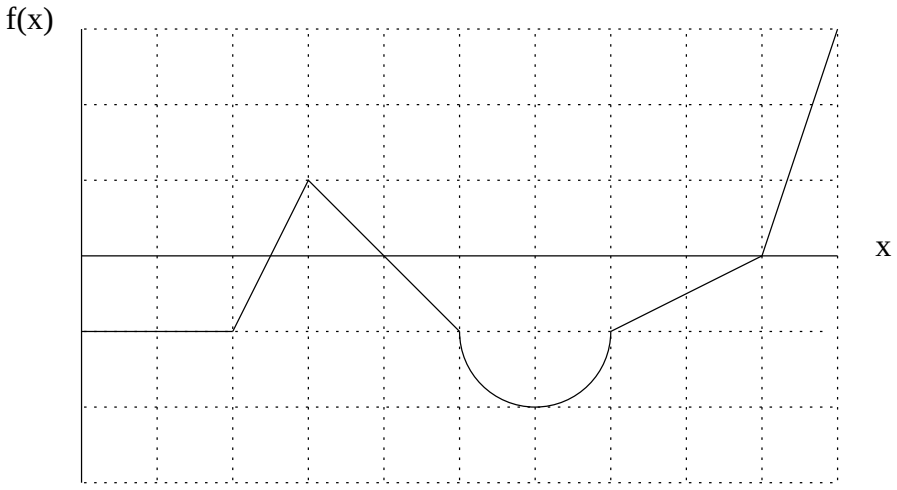
$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

to give the best upper bound you can on the error in the approximation T_4.

(e) How large must n for M_n to approximate $\int_0^4 x^2 dx$ to within .001?

5. Below is the graph of a function $f(x)$, continuous on the interval $[0, 10]$. Define a new function $g(x)$ by

$$g(x) = \int_0^x f(t)dt$$



(a) On what intervals is $g(x)$ increasing?

(b) Compute $g(7)$.

(c) Compute $g'(3)$.

(d) List the x-values of any local minima of $g(x)$, or write "NONE" if there are none.

(e) On what intervals is $g(x)$ concave down?

(f) List the x-values of any inflection points of $g(x)$, or write "NONE" if there are none.

(g) What is the average value of $f(x)$ on the interval $[0, 10]$?

6. **(a)** Find the area of the region enclosed by the curves $y = x^2$ and $y = x^3$.

(b) Find the volume of the solid created when the above region is rotated around the line $x = -1$.

(c) Find the volume of the solid created when the above region is rotated around the line $y = -2$.

(d) Find the volume of the solid whose base is the above region and with cross sections perpendicular to the x-axis squares.

7. Define a function $h(x)$ by:

$$h(x) = \int_{4x}^{\sin(2x)} \frac{\cos(t)}{t} dt$$

Find $h'(x)$.

8. Write down a definite integral which computes the length of the curve $y = \sin(x)$ between $x = 0$ and $x = \pi$.

9. Suppose that $\rho(x)$ represents the density of a thin rod in kg/cm, where x is measured in cm. What does

$$\int_5^{10} \rho(x) dx$$

represent? Include units.

10. A function $f(x)$ is given by:

$$f(x) = \begin{cases} 0 & x < 1 \\ \frac{k}{x^4} & x \geq 1 \end{cases}$$

(a) For what value of k is $f(x)$ a probability density function?

(b) What is the mean of the random variable X associated to the probability density function $f(x)$?

(c) What is the median of the random variable X associated to the probability density function $f(x)$?

11. A hemispherical tank is 10 meters deep, and is full of water. (The tank looks like the bottom half of a sphere.) How much work does it take to pump all of the water out of the top of tank? (Water has a density of 1000 $\frac{kg}{m^3}$.)

12. A car has initial velocity 10 ft/s and accelerating at a rate of 12 ft/s^2. How far does the car travel in 3 seconds?