MAT 125 Summer II 2008

Practice Final Exam

There are 12 questions on this practice exam. The actual exam will have 8 questions, each similar but not idential to one of the 12 questions on this practice final. For example, this practice final has a related rates problem involving a cylinder. Thus there may be a related rates problem on the final, but not necessarily involving a cylinder.

In many cases, the problems on the final will be shorter, and have fewer parts, then the questions on this practice exam.

Solutions may appear on the course webpage.

1. Below are some statements about functions. If the statement is *always true* no matter what the function is, cicle "TRUE". If the statement is *sometimes true and sometimes false*, depending on what the function is, circle "FALSE".

(a) If $f(x)$ is continuous on the open interval (a, b), then $f(x)$ has an abolute maximum on the interval (a, b)

TRUE FALSE

(b) If $g(x)$ has a local maximum at $x = 3$ then $g'(3) = 0$

TRUE FALSE

(c) If $g'(3) = 0$, then $g(x)$ has a local maximum at $x = 3$

TRUE FALSE

(d) If $h(x)$ is differentiable on the interval $[1, 3]$, $h(1) = 3$ and $h(2) = 7$, then for some number c between 1 and 3, $h'(c) = 2$

TRUE FALSE

(e) If $\lim_{x \to 3} \frac{f(x)-f(3)}{x-3} = 17$, then $f(x)$ is continuous at $x = 3$

TRUE FALSE

(f) If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$

TRUE FALSE

(g) If $g(x)$ is a 1-to-1 dfferentiable function, then $g^{-1}(x)$ exists and is differentiable

TRUE FALSE

(h) If $f(x)$ is a continuous function and a and b are real numbers in the domain of $f(x)$, then $f(a+b) = f(a) + f(b)$

TRUE FALSE

(i) If $f(x)$ is a continuous function, $f(1) = 4$, and $f(3) = 12$, then $4 \leq f(2) \leq 12$

TRUE FALSE

(j) The graph of a continuous function can cross a vertical assymtope of the function

TRUE FALSE

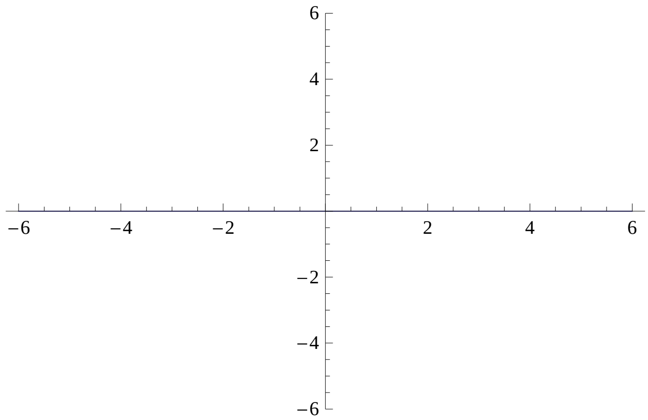
(k) The graph of a continuous function can cross a horizontal assymtope of the function

TRUE FALSE

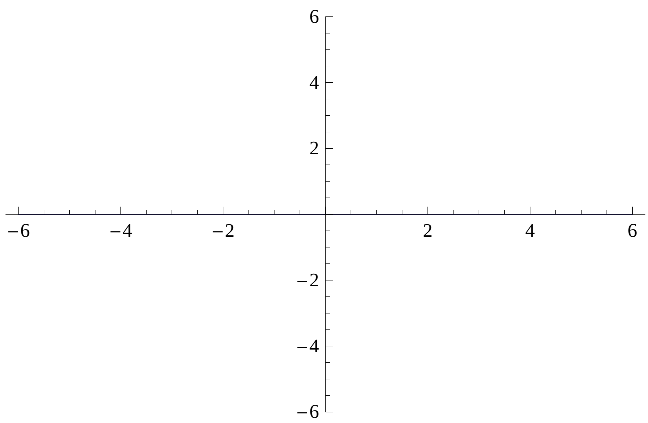
(l) If $\lim_{x\to 7} f(x)g(x) = 24$, then $\lim_{x\to 7} f(x)$ exists

TRUE FALSE

2. Consider the function $f(x) = 4 - e^x$
(a) Sketch $f(x)$ on the axes provided, and state the domain and range of $f(x)$. Clearly indicate horitontal and vertical assymtopes on your graph.



(b) Write a formula for the inverse function of $f(x)$. Then sketch a graph of $f^{-1}(x)$ and state its domain and range.



3. Compute the following limits. If the limit exists and is a real number, write the number. If a limit does not exist, write "$+\infty$", "$-\infty$", or "DNE" as appropriate. Do not write "undefined", because I won't know what you mean.

(a) $\lim_{x \to \frac{\pi}{2}} e^{\cos x}$

(b) $\lim_{x \to 2} \frac{x-3}{x^2-9}$

(c) $\lim_{x \to 3} \frac{x-3}{x^2-9}$

(d) $\lim_{x \to \infty} \frac{x-3}{x^2-9}$

(e) $\lim_{t \to 7} \frac{t-2}{t^2-8t+7}$

(f) $\lim_{x\to\infty} \sin(x)$

(g) $\lim_{h\to 0} \frac{\sqrt{h+4}-2}{h}$

(h) $\lim_{y\to\infty} \sqrt{y^2+7y+2} - y$

(i) $\lim_{u\to\infty} -\ln(u)$

(j) $\lim_{t\to\frac{\pi}{2}^+} e^{\tan(t)}$

The limits on this page are trickier. Try the Squeeze Theorem and l'Hospital's rule.

(k) $\lim_{x\to 0} \frac{\cos(x)-1}{x}$

(l) $\lim_{x\to 0} x^2 \sin(\frac{1}{x})$

(m) $\lim_{x\to 0^+} \ln(x)\sin(2x)$

(n) $\lim_{x\to \infty} e^{-x}\sin(x)$

(o) $\lim_{x\to 6^+} \frac{2x-12}{|x-6|}$

4. Differentiate the following funtions with respect to x.

(a) $(7x^7 - 6x^6)^2$

(b) $2\cos(2x)$

(c) xe^{-3x}

(d) $\frac{\ln(x)}{11x^3}$

(e) $\sin(\sqrt{1 + e^x})$

(f) $\sec(x^2)$

(g) e^3

(h) $x^4 + 4^x + \sin^{-1}(4x)$

(i) $\log_2(\sqrt[3]{x})$

(j) $\frac{3x - xe^{2x}}{\tan(x)}$

5. Find $\frac{dy}{dx}$ in each of the following problems:

(a) $y = e^{-x\sin(x^2)}\cos(\sqrt{7x^3 - 3x})$

(b) $xy^3 - yx^3 = 2$

(c) $ye^{x+y} = 10x$

6. $f(x)$ and $g(x)$ are differentiable functions such that $f(2) = 3$, $f'(2) = 5$, $g(2) = -2$, $g'(2) = 4$, $f'(-2) = 4$, and $f'(4) = -1$. Compute the following derivatives:

(a) $h(x) = f(x)g(x)$ Find $h'(2)$

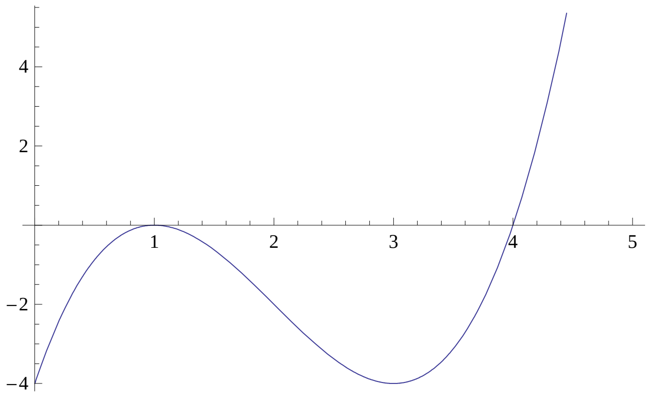
(b) $h(x) = f(g(x))$ Find $h'(2)$

(c) $h(x) = \ln(f(x))$ Find $h'(2)$

(d) $h(x) = \frac{x^2}{g(x)}$ Find $h'(2)$

(e) $h(x) = 3[g(x)]^2 + 7[f(x^2)]$ Find $h'(2)$

7. $f(x)$ is a differentiable function defined on the interval $[0, 4.5]$. Here is a graph of $f'(x)$, the **derivative** of $f(x)$. Answer the following questions about $f(x)$.



(a) On what intervals is $f(x)$ increasing?

(b) On what intervals is $f(x)$ decreasing?

(c) List the x-values of any local maxima of $f(x)$, or write "NONE" if there are none.

(d) List the x-values of any local minima of $f(x)$, or write "NONE" if there are none.

(e) On what intervals is $f(x)$ concave up?

(f) On what intervals is $f(x)$ concave down?

(g) List the x-values of any inflection points of $f(x)$, or write "NONE" if there are none.

8. Consider $g(x) = \frac{x^4}{4} - x^3 + x^2$

(a) On what intervals is $g(x)$ increasing?

(b) List the local minima of $g(x)$, or write "NONE" if there are none

(c) Find the absolute maximum and minimum values of $g(x)$ on the interval $[0, 2]$

(d) List the inflection points of $g(x)$

(e) Where is $g(x)$ concave up?

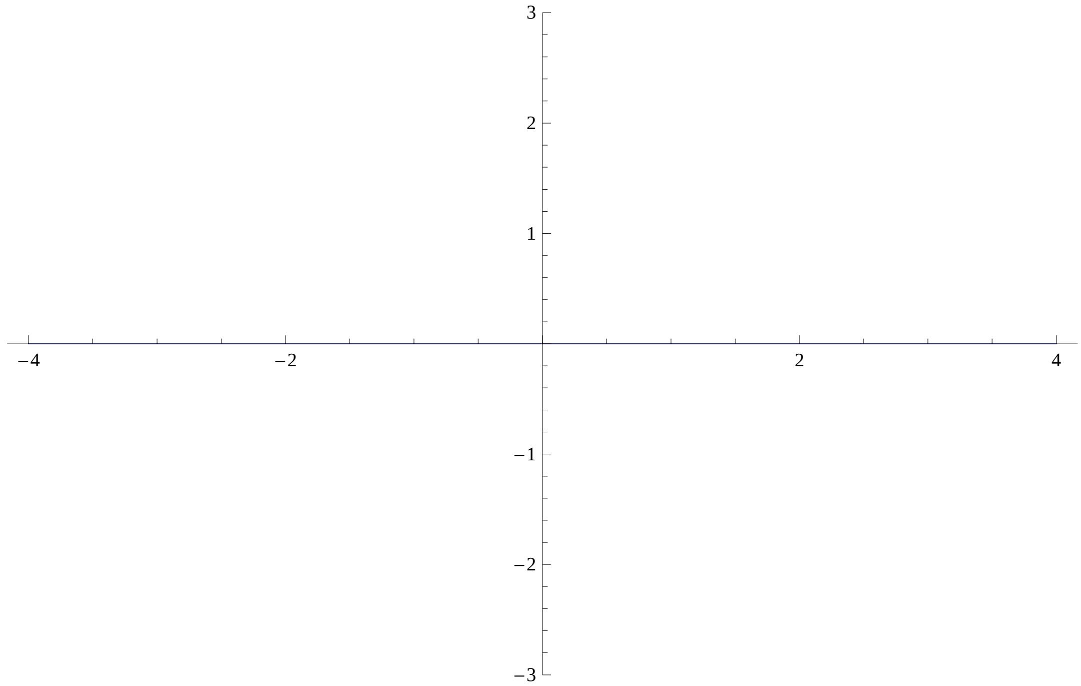
9. Let $f(x) = \sqrt{x}$

(a) Find the equation of the tangent line to $f(x)$ at $x = 4$

(b) Use your from part (a) to approximate $\sqrt{3.9}$

10. Sketch the graph of a function $f(x)$ satisfying the properties listed below. Make sure your graph is the graph of a function!

- $f(0) = 0$
- $f'(x) > 0$ on $(-1, 1)$
- $f'(x) < 0$ on $(-\infty, -1)$ and $(1, \infty)$
- $f'(1) = f'(-1) = 0$
- $f''(x) > 0$ on $(-2, 0)$ and $(2, \infty)$
- $f''(x) < 0$ on $(-\infty, -2)$ and $(0, 2)$
- $f''(2) = f''(-2) = 0$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 0$



11. Oil oozes out of the bottom of truck into a 2 inch deep cylindrical puddle at a rate of 5 cubic inches per minute. At what rate is the radius of the puddle increasing when the radius is 12 inches? (Recall that the volume of a cycliner of radius r and height h is given by the formula $V = \pi r^2 h$.)

12. Find the area of the largest rectangle with two vertices on the x-axis and two vertices on the graph of the parabola $y = 9 - x^2$.