

MY NAME IS: \_\_\_\_\_

1	2	3	4	5	6	7	8	9	10	Total

MAT 126  
Calculus B  
Final  
August 16, 2001

SHOW ALL YOUR WORK ON THESE PAGES! TOTAL SCORE = 120

**Table of indefinite integrals**

$$\left. \begin{array}{l} \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \\ \int e^x dx = e^x + C \\ \int \cos x dx = \sin x + C \end{array} \right| \begin{array}{l} \int \frac{1}{x} dx = \ln|x| + C \\ \int \sin x dx = -\cos x + C \\ \int \frac{1}{1+x^2} dx = \tan^{-1}x + C \end{array}$$

1. Evaluate the definite integral  $\int_1^e \frac{(\ln x)^2}{x} dx$ .

2. Evaluate the following indefinite integrals.

(a)  $\int \sin^2 x \cos^3 x \, dx$

(b)  $\int \frac{x}{x^2 - 4x + 5} \, dx$

3. Approximate  $\int_0^1 \sqrt{1+x^3} dx$  using Trapezoid rule and Simpson's rule with 4 subintervals. (You could write out the expression explicitly without calculating the decimal value.)

4. Compute the exact value of the infinite integral  $\int_0^\infty xe^{-x} dx$ .

5. Use the comparison test to tell if the following improper integrals are convergent or divergent.

(a)  $\int_1^{\infty} \frac{1}{\sin^2 x + e^x} dx$

(b)  $\int_2^{\infty} \frac{1}{\sqrt[3]{x-1}} dx$

6. Find the area of the region bounded by the graphs of  $x = y^2 - 3$  and  $x = 2y$ .

7. Consider the planar region bound by  $y = \sqrt{x}$ ,  $y = 0$  and  $x = 1$ .

(a) Rotate this region about the x-axis, compute the volume of the solid obtained.

(b) Rotate this region about the y-axis, compute the volume of the solid obtained.

8. Find the arc length of the curve  $x = t^4$ ,  $y = t^6$  from point  $(0,0)$  to point  $(1,1)$ .

9. The temperature  $T$  of a cup of coffee (in degree Celsius),  $t$  hours after brewing, is  $T(t) = 25 + 75e^{-2t}$ . What is the average temperature of the coffee in the first half hour?

10. A vertical water tank is in the shape of a circular cylinder. The height of the cylinder is 4 meters, and the radius of the base circle is 1 meter. The tank is full of water. How much work is done pumping all the water out to the top of the tank?  
(The gravitational constant is  $g = 10m/s^2$ , and the density of water is  $1000kg/m^3$ .)