Geometric [illegible]

[illegible]:

$P_0 + RP_0 + R^2 P_0 + \dots + P_0 R^{n-1} = \frac{P_0 - P_0 R^n}{1 - R}$

Unless $R = 1$
R common ratio

Recall $P_0 + \dots + P_0 R^{n-1} = [illegible]$

find [illegible]

Example $1 + 2 + 4 + 8 + 16 + 32 + 64$

$2^0 + 2^1 + \dots + 2^6 = \frac{1 - 2^7}{1 - 2}$

$2 + 4 + 8 + 16 + 32 + 64 + 128 = \frac{2 - 256}{1 - 2} = 254$

[illegible] la formule

$$1 + 2 + 4 + 8 + 16 + 32 + 64 = S \quad (L_1)$$

$$2 + 4 + 8 + 16 + 32 + 64 + 128 = 2S \quad (L_2)$$

$$(L_1) - (L_2) = \cancel{128} - 1 = 1 - 128$$

$$(L_1) - (L_2) = S - 2S = (1 - 2) \times S$$

$$(1 - 2) \times S = 1 - 128$$

$$S = \frac{1 - 128}{1 - 2}$$

$$R = 1 \qquad P_0 + P_0 + \dots + P_0 R^{n-1}$$

$$P_0 + \dots + P_0 = P_0 n$$

$\Sigma_2 /$

Bsp

$2 + 4 + 8 + 16 + \dots + 128 = S$

$(\times 2)$ $4 + 8 + 16 + \dots + 128 + 256 = 2S$

$2 - 256 = S - 2S$

$\frac{2 - 256}{1 - 2} = S$

Bsp

$6 + 12 + 24 + \dots +$

$3\times 2 + 3\times 2^2 + \dots + 3\times 2^5 = S$

$(\times 2)$ $3\times 2^2 + \dots + 3\times 2^5 + 3\times 2^6 = 2S$

$3\times 2 - 3\times 2^6 = (1-2)S$

$\frac{3\times 2 - 3\times 2^6}{1 - 2} = S$

$P_0R^k + P_0R^{k+1} + \dots + P_0R^n = S$

$P_0R^{k+1} + \dots + P_0R^n + P_0R^{n+1} = RS$

$P_0R^k - P_0R^{n+1} = S - RS = (1-R)S$

$$\boxed{\frac{P_0R^k - P_0R^{n+1}}{(1-R)} = S}$$

Example 6.3 P_n geometric sequence

$P_0 = 4$ $P_1 = 6$ $P_2 = 9$

- Find the common ratio $R = \frac{P_1}{P_0} = \frac{6}{4} = \frac{3}{2}$
- Compute $P_0 + P_1 + \dots + P_{10}$

$P_n = 4 \cdot \left(\frac{3}{2}\right)^n$

$4 + 6 + 9 + \dots + P_{10}$

$4 + 4 \cdot \frac{3}{2} + \dots + 4 \cdot \left(\frac{3}{2}\right)^{10}$

$$\boxed{S = \frac{4 - 4\left(\frac{3}{2}\right)^{11}}{1 - \left(\frac{3}{2}\right)}}$$

39 $P_n = 4P_{n-1}$ with initial population $P_0 = 5$.

- Find P_1, P_2, P_3 $P_1 = 20$ $P_2 = 80$ $P_3 = 320$
- Give a formula for P_n $P_n = 4^n \cdot 5$
- How many generations will it take to reach 1 million?

$P_n \geq 1000000$ $4^n \cdot 5 = 10^6$ $4^n = 2 \cdot 10^5$

Sayfa V

3)

$4^{n-1} \cdot 2 = 10^5$

$4^{n-1} = 5 \cdot 10^4$

$4^{n-1} = 50000$

$n=1 \rightarrow 4^{n-1} = 1$

$n=2$	4
3	16
4	64
5	256
6	1024
7	4096
8	16384
9	65536

$\boxed{n = 9}$

Arithmetic sequences

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Gauss method

Compute $1+2+3+\dots+99+100$

$$\boxed{\begin{matrix}1\\100\end{matrix}} \begin{matrix}+2+3+4+5+\dots+100=S\\+99+\dots+1=S\end{matrix}$$

$$\underbrace{101 \quad 101 \quad 101 \quad \dots \quad 101}_{100 \text{ terms (columns)}}$$

$101+101\dots+101=S+S$

$100\times101=2S$

$$\boxed{S=\frac{100\times101}{2}}$$

Compute $\underbrace{51 + 52 + \dots + 90}_{40\ \text{terms}} = S$

$51 + 52 + \dots + 90 = S$
$90 + 89 + \dots + 51 = S$
141

$141 \cdot 40 = 2S$

$$\boxed{S = \frac{141 \cdot 40}{2}}$$

Do I know by heart the formula?

Compute $0 + 2 + 4 + 6 + \dots + 100 = S$

$0 + 2 + 4 + \dots + 100 = S$
$100 + 98 + \dots + 0 = S$
$\underbrace{100}_{50\ \text{times}}$

$2S = 50 \cdot 100 = 5000$

$$\boxed{S = 2500}$$

Sequence III
2/

Gauss[illegible]

u_n = arithmetic sequence

$u_0 = I \quad u_n = u_{n-1} + d$

Compute $u_0 + u_1 + \dots + u_n$?

Apply Gauss Trick $u_n = I + nd$

$$\underbrace{\begin{matrix} (I + 0 \times d) + (I + 1d) + \dots + (I + nd) = S \\ (I + nd) + (I + (n-1)d) + \dots + (I + 0d) = S \\ \downarrow \qquad\qquad \downarrow \\ (2I + nd) \quad (2I + nd) \end{matrix}}_{n+1 \text{ terms}}$$

$$2S = (n+1) \times (2I + nd)$$

$$S = (n+1)\left[I + \frac{nd}{2}\right]$$

$$S = \frac{(n+1)(2I + nd)}{2}$$

Compute $u_2 + u_3 + \dots + u_{10}$?

$S = u_2 + u_3 + \dots + u_{10}$

$u_n = I + nd$

$S = (I + 2d) + \dots + (I + 10d)$

$S = (I + 10d) + \dots + (I + 2d)$

$\downarrow$

$(2I + 12d)$

$2S = (2I + 12d) \times 9$

$S = \frac{(2I + 12d)}{2} \times 9 = (I + 6d) \times 9$

Sequence 3/

Let u_n be the sequence def by
Compute $u_1 = 1$

$u_{n+1} = u_n - 3$

Compute $u_1 + u_2 + \dots + u_{100}$?

* Step 1 Find the formula for u_n (Template Day 1!)
* Step 2 Apply Gauss trick
  [Sum of pairs of terms, but number of terms]

Conclude

Step 1 $u_n = \blacksquare + \blacksquare \times \blacksquare$

slope $d = -3$

$u_n = \blacksquare - 3n$

$\boxed{u_n = 4 - 3n}$

Step 2 $a_3 + a_4 + \dots + a_{100} = \underline{\underline{S}}$

$$(4-3\times3)+(4-3\times4)+ \dots +(4-3\times100) = S$$

$$(4-3\times100)+ \dots +(4-3\times3) = S$$

$$\downarrow$$

$$\underbrace{(4-3\times103)}_{100-3+1 \text{ terms}}$$

$$S = \frac{98\times(4-3\times103)}{2}$$

$\boxed{\text{Logistic growth model}}$

Week 13
I 1/

- the recursive formula

$$p_{n+1} = r(1-p_n)p_n$$

r = growth parameter

p_n percentage of the population at stage n

What it models?

Suppose you have a population living in an area with fixed space

here maximal occupancy = 27.

Start with an initial population I (ex: I=3)
P_0

Growth of the population depends on the empty space

Here amount of empty space at day 0

$$27 - I = 27 - P_0 \quad \text{i.e. fraction } \frac{27-P_0}{27}$$

Growth rate $= \dfrac{(27-P_0)}{27 \text{ empty space}} \times r$

$$\rightsquigarrow \quad P_1 = \underbrace{\left(\frac{27-P_0}{27}\right) \times r}_{\text{growth rate}} \times P_0 \qquad (*)$$

If we set $p_1 = \dfrac{P_1}{27}$ fraction of occupancy at day 1

$$p_0 = \frac{P_0}{27}$$

$(*)$ writes as

$$p_1 = \frac{P_1}{27} = \frac{27 - P_0}{27} \times r \times \frac{P_0}{27}$$

$$p_1 = (1 - p_0) \times r \times p_0$$

Ibid % empty given $1-p_n$ of day 1
$r(1-p_n)$ growth of day 1

$p_2 = r(1-p_1) \times p_1$

etc....

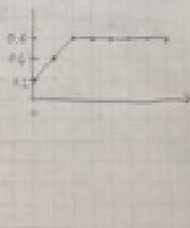
What can happen?

$p_0 = 0.2 \quad r = 2.5$

$p_1 = 2.5(1-0.2) \times 0.2$

$= 2.5 \cdot \frac{4}{5} \cdot \frac{1}{5}$

$= \frac{0.5 \times 4}{5} = \frac{2}{5} = 0.4$



$p_2 = 2.5(1-0.4)(0.4)$

$= 2.5 \cdot \frac{3}{5} \cdot \frac{2}{5}$

$= 0.6$

$p_3 = 2.5(1-0.6)\,0.6$

$= 2.5 \quad 0.4 \times 0.6$

$= 0.6$

second example,

$p_0 = 0.3 \quad r = 2.5$

$p_1 = 2.5(1-0.3)0.3$

$= 2.5 \cdot 0.7 \cdot 0.3$

$= 0.525$

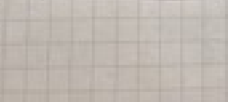
$p_2 = 2.5 \times (1-0.525)\, 0.525 \simeq 0.6234$

$p_3 \simeq 2.5 \times (1-0.6234) \times 0.6234 \simeq 0.5869$

$p_4 \simeq 2.5 \times (1-0.5869) \times 0.5869 \simeq 0.6061$

$p_5 \simeq 2.5 \times (1-0.6061)\, 0.6061 \simeq 0.5968$

$p_6 \simeq 2.5 \times (1-0.5968)\, 0.5968 \simeq 0.6016$



Week 13
I 9/

Example

$p_0 = 1 - 0.3$.

$p_1 = 0.525$

$p_2 \simeq 0.6236$

Two cycles (cobweb).

$\lambda = 3.1$ $p_0 = 0.2$

$p_1 = 0.2$ $p_1 \simeq 0.696$ $p_1 \simeq 0.775$

$p_3 \simeq 0.561$ $p_4 \simeq 0.770$

$p_5 \simeq 0.569$ $p_6 \simeq 0.767$ $p_7 \simeq 0.553$.

$p_8 \simeq 0.766$ $p_9 \simeq 0.555$ $p_{10} \simeq 0.766$

$p_{11} \simeq 0.556$ $p_{12} \simeq 0.765$. $p_{13} \simeq 0.559$ $p_{14} \simeq 0.765$

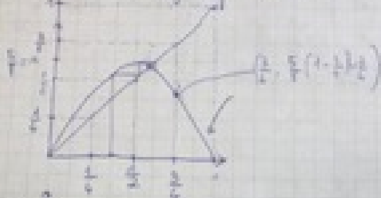
$p_{15} \simeq 0.557$.

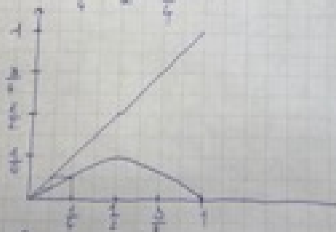
4 cycle: $\lambda = 3.5$

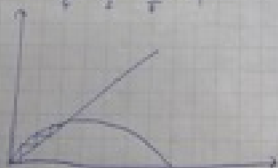
$p_0 = 0.875$ $p_1 = 0.383$. $p_{23} \simeq 0.821$ $p_{24} \simeq 0.501$

$p_{25} \simeq 0.875$.

logistic growth model:





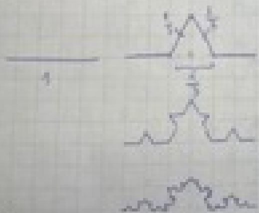


The behavior for different [illegible] is difficult to predict
and can be very different
$\Rightarrow$ it is worth [illegible] [illegible] ([illegible])

Other types of fractals?

Von Koch snowflake

Rule: separate each segment
replace by taking the middle segment
→ by two sides of the same length



What is the length of the snowflake at stage n?
What is the area of the snowflake at stage n?

step 1   1 segment   length 1

step 2 $4 \times \frac{1}{3}$

step 3 $4 \times 4 \times \frac{1}{9}$

step 4 $4 \times 4 \times 4 \times \frac{1}{27}$

At step n $\left(\frac{4}{3}\right)^{n-1}$ with length

What happens for the area

step 1   0

step 2 $\frac{1}{2} \times \frac{1}{3} \times \sqrt{\left(\frac{1}{3}\right)^2 - \left(\frac{1}{6}\right)^2}$

$= \frac{1}{6} \times \frac{\sqrt{3}}{6\sqrt{3}}$

$\frac{\sqrt{3}}{36}$