Population growth

Aka sequences.

<u>Most important definition.</u>

Sequence = list of numbers. (infinite list of numbers).

<u>Example</u>:

		After
$\boxed{A}$	1, 2, 3, 4, 5, 6, 7 . . .	$u_1 = 1$ $u_2 = 2$ $u_n = n$.
$\boxed{B}$	0, 2, 4, 6, 8, ~~10~~ 10, 12, . .	$u_1 = 2$ $u_2 = 4$ $u_n = 2 \times n$.
$\boxed{C}$	1, -1, 1, -1, 1, -1, 1, -1, . .	$u_1 = 1$ $u_2 = -1$ $u_n = (-1)^{n+1}$

How does all sequences look like?
How to work with them?

General sequence ($n_1, n_2, n_3, n_4, \ldots$

People usually write $u_1, u_2, u_3, \ldots$,

quick writing (u_n)

1

Work with a general formula

the element number n (of rank n) u_n is determined by a formula depending on n

$u_n = n$ $\qquad$ $u_n = 2^n$.

$u_n = 2n$ $\qquad$ $u_n = (-1)^n$.

$u_n = n^2$

Why do we study sequences?

- Every day I look at the number of persons on this planet.
- Every second I record the level of carbon dioxyde in the atmosphere,
- Every day, I look at the market value of HBO
- Count the number of radioactive particles.

Famous example: (self procreating rabbits).

- start with 1 rabbit in day 1.
- ~~day~~ 2, it becomes fertile.

every month each pair of rabbits bears a new pair which from the second month becomes productive.

1st month	1 Pair	$P_1 = 1$
2nd month	1 Pair	$P_2 = 1.$
3rd month	2 Pair	$P_3 = 2$
4th month	3 Pair	$P_4 = 3.$

recursive rule $P_n = P_{n-1} + P_{n-2}$
with $P_0 = P_1 = 1.$

This is called the Fibonacci sequence. (1175 – 1250).

He introduced the arabic notation for computation in Europe

question: Can you predict the behavior of the sequence?

Is there a formula for P_N depending on N?

$$R_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} + \left(1 - \frac{1+\sqrt{5}}{2\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^{n}$$

$$= \frac{1}{\sqrt{5}}\left[\frac{\sqrt{5}-1}{2}\right]\left(\frac{1-\sqrt{5}}{2}\right)^{n}.$$

$$\boxed{R_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n\cancel{+1}} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{n\cancel{+1}}}$$

This sequence is already very advanced.
Its growth is controlled by a particular number:

$\frac{1+\sqrt{5}}{2}$ is the golden ratio. (Renaissance called it the divine proportion).

$\shortparallel$

$1{,}61803\ldots$

Wikipedia says:

- Appears in the Greek construction of the regular pentagon.
- flower head of the sunflower.
- Used in monuments and art pieces (by Corbusier, some paintings of Dali).
- Bombieri – Cardano worked on it
- Kepler later studied them.

9.1 Ex 9, 18
9.2 Ex 34, 19

Population growth II

Example:

Suppose a sequence A_N is given by.

$$A_n = ~~k~~ n^2 + 1.$$

- Find A_1
- Find A_{100}
- Suppose $A_N = 10$ Find N

A_1 Replace n by 1.

$$A_{\boxed{1}} = \boxed{1}^2 + 1 \quad \text{(with colors)}.$$

$$A_{\boxed{100}} = \boxed{100}^2 + 1$$

$$= 10000 + 1 = 10001.$$

$A_N = 10$ N?

Write what you know $A_N = \underline{\underline{N}}^2 + 1 = 10$

$$\underline{\underline{N}}^2 = 10 - 1 = 9. \quad \boxed{N = 3}$$

Example: $A_n = \dfrac{4n}{n+3}$

Find A_1 ? $\quad A_{\boxed{1}} = \dfrac{4 \times \boxed{1}}{\boxed{1}+3} = 1.$

" A_9 ? $\quad A_{\boxed{9}} = \dfrac{4 \times \boxed{9}}{\boxed{9}+3} = \dfrac{36}{12} = 3.$

$A_N = \dfrac{5}{2}$ Find N.

$A_N = \dfrac{4N}{N+3} = \dfrac{5}{2}$. $\quad 4N = \dfrac{5}{2}(N-3)$

TO DO Isolate N

$$4N = \frac{5N}{2} - \frac{5}{2} \times 3$$

$$\left(4 - \frac{5}{2}\right) N = -\frac{15}{2}$$

$$\boxed{N = -5.}$$

Example

$$A_N = 2A_{N-1} + A_{N-2}$$

$A_1 = 1 \quad A_2 = 1.$

Find A_8

$A_1 = 1 \quad A_2 = 1 \quad A_3 = 2 \times 1 + 1 = 3.$

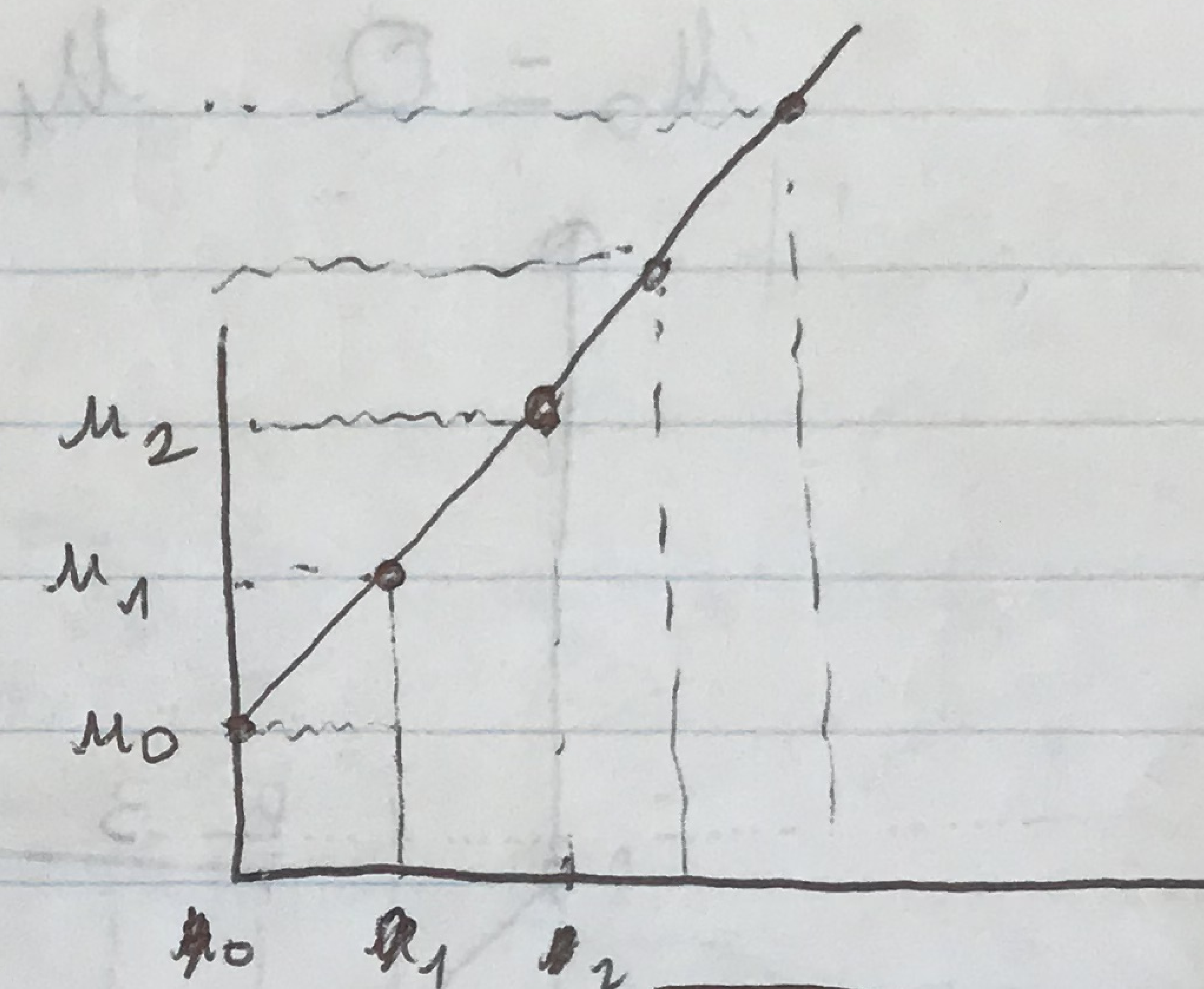
$A_4 = 2 \times 3 + 1 = 7 \quad A_5 = 2 \times 7 + 3 = 17$

$A_6 = 2 \times 17 + 7 = 41 \quad A_7 = 99$

$$\boxed{A_8 = 198 + 41 = 239}$$

Linear growth

Example: every day, the sequence increases by the same quantity.



quantity $= 1$.

$u_0 = 1 \quad u_1 = 1 + 1 = 2$

$u_2 = 2 + 1 = 3 \quad u_3 = 3 + 1 = 4$ $\boxed{u_n = n + 1}$

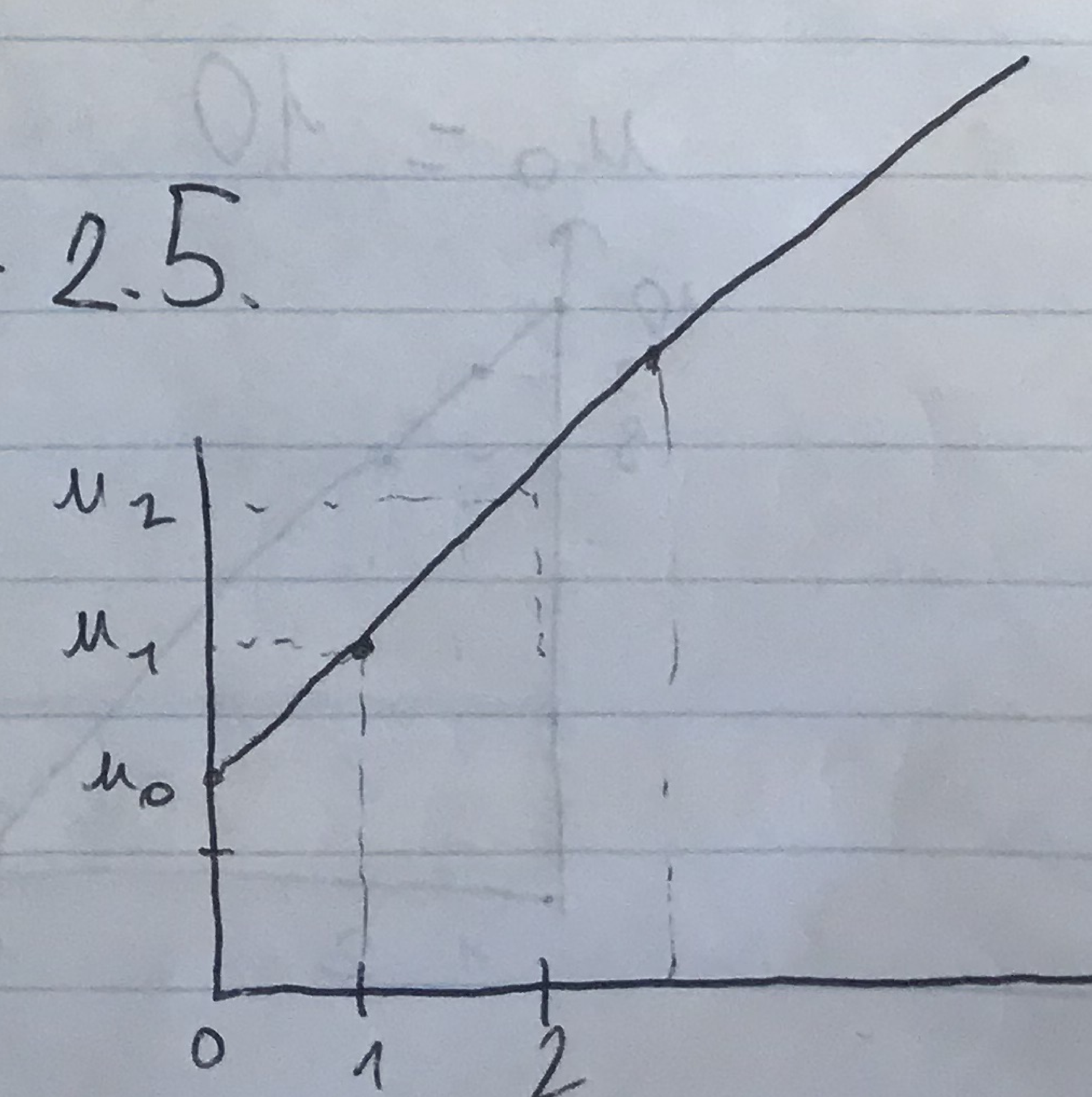
$n = 2 \quad n + 1 = 3.$

Guess the general formula?

$u_0 = 1.5 \quad u_1 = 1.5 + 1 = 2.5.$

$u_2 = 3.5$

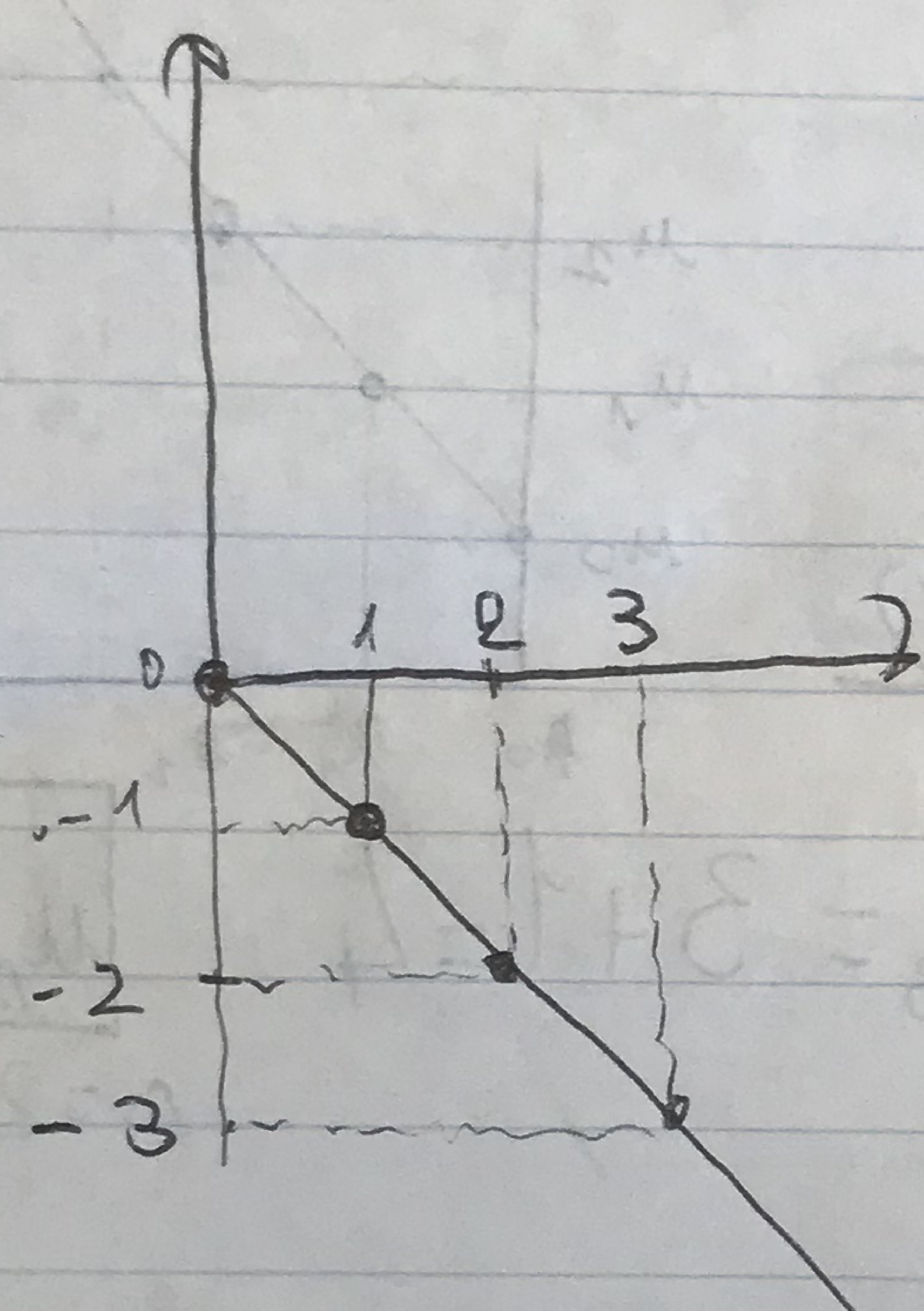
$\boxed{u_n = 1.5 + n}$



Decreasing case:

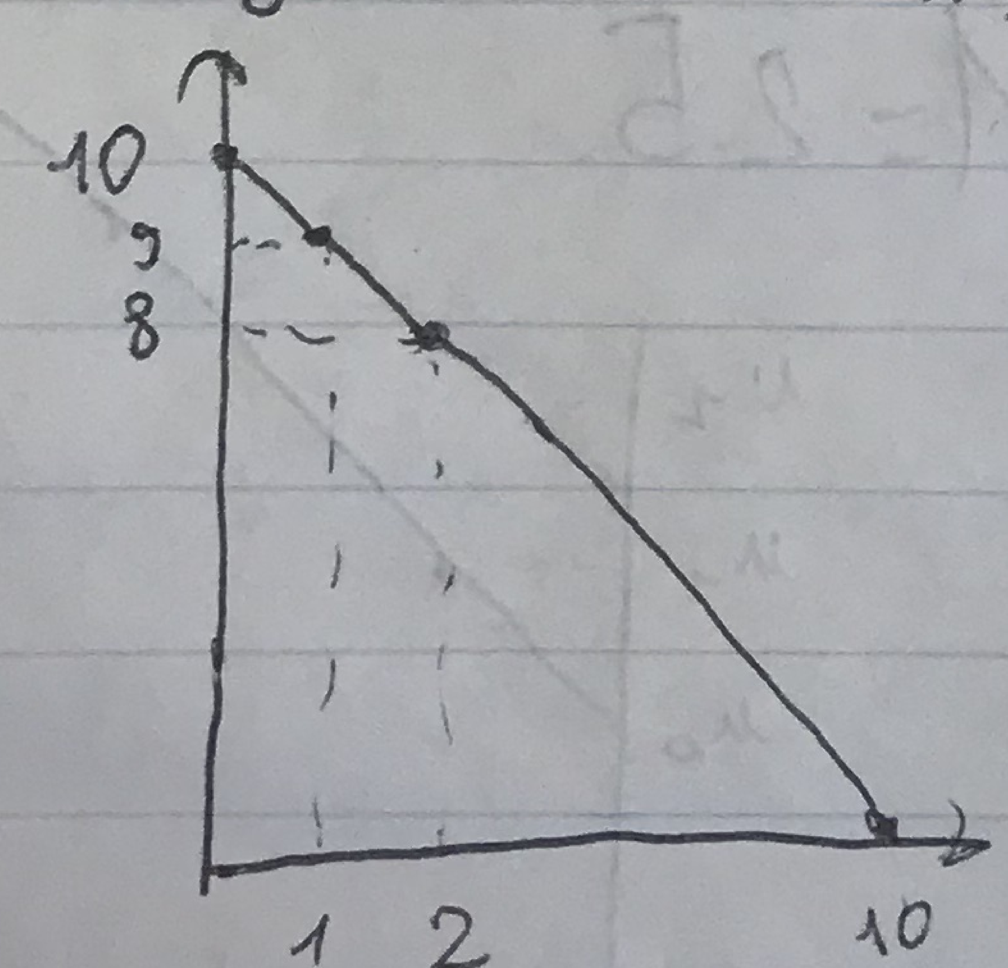
by 1, by 2.

$u_0 = 0 \quad u_1 = -1 \quad u_2 = -2, \ldots$



$\boxed{u_n = -n}$

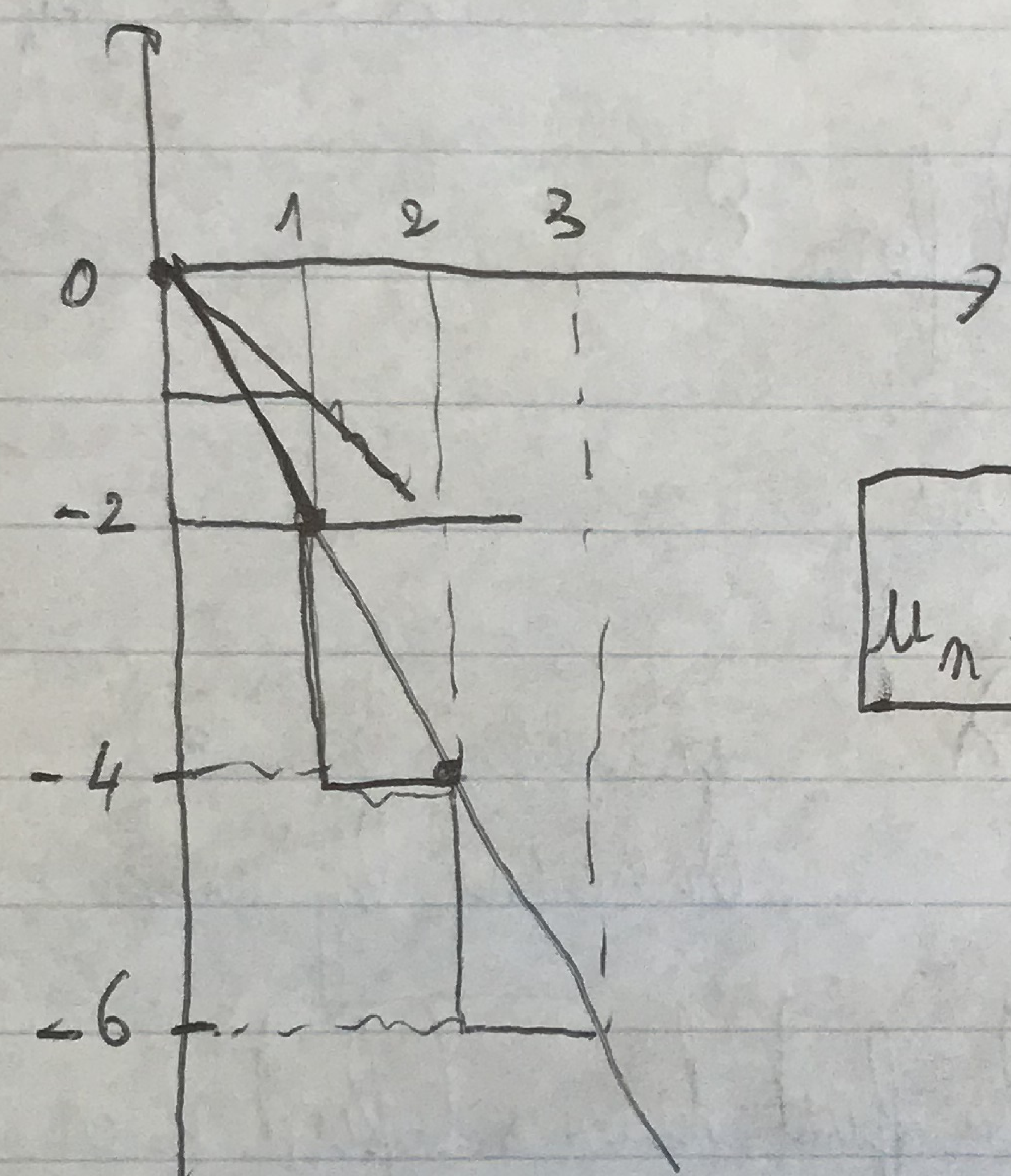
$u_0 = 10 \quad u_1 = 9 \quad u_2 = 8.$



$\boxed{u_n = 10 - n}$.

Starting by 0 decreasing by 2.

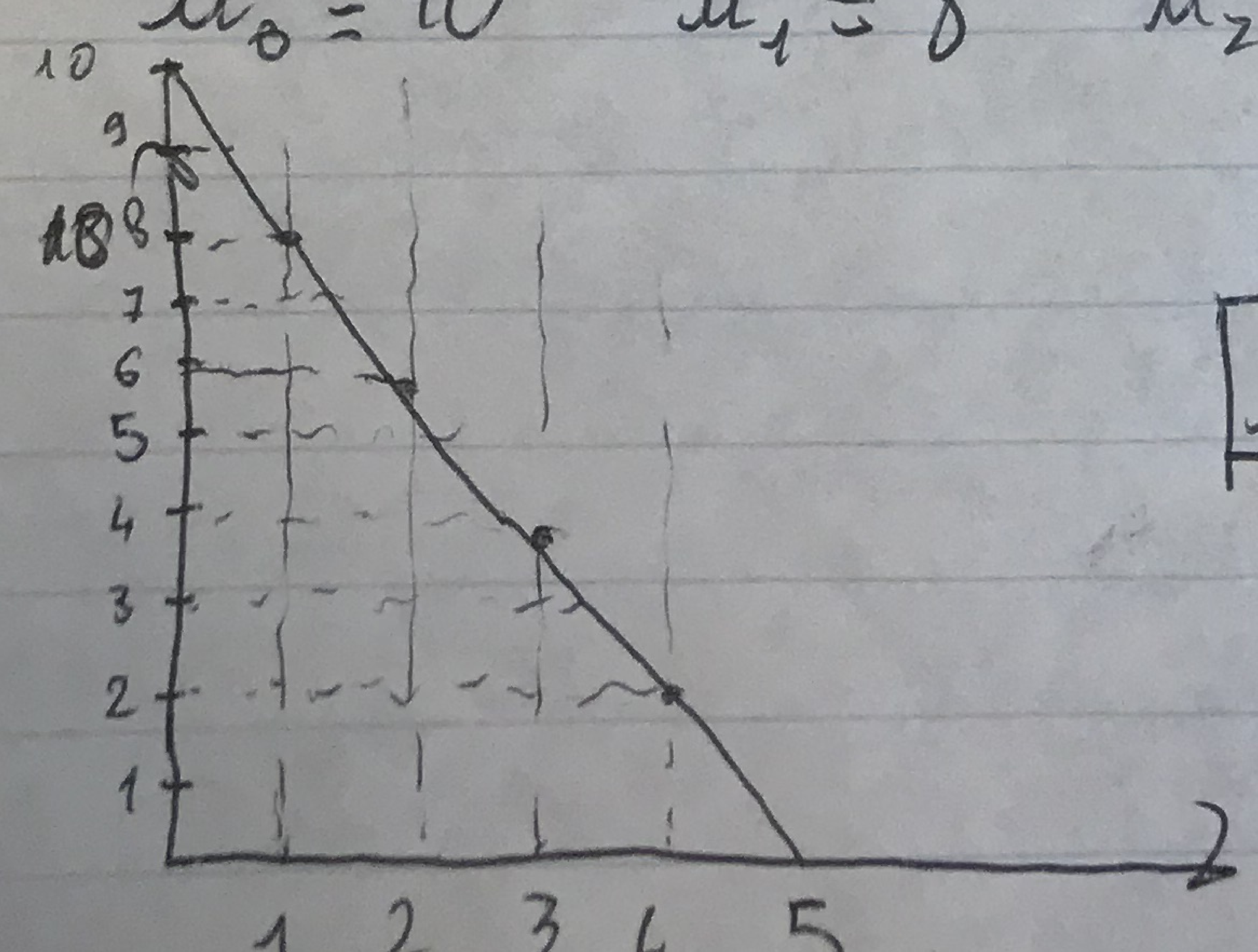
$u_0 = 0 \quad u_1 = -2 \quad u_2 = -4, \; u_3 = -6.$



$\boxed{u_n = -2n}$

Starting by 10 decreasing by 2.

$u_0 = 10 \quad u_1 = 8 \quad u_2 = 6 \quad u_3 = 4.$



$\boxed{u_n = 10 - 2n}$

General form

$\boxed{u_n = a + \lambda n}$ ~~decr~~ $\lambda > 0$ increasing.
$\lambda < 0$ decreasing.

How to guess the general form?

Step 1 • Compute the first terms.

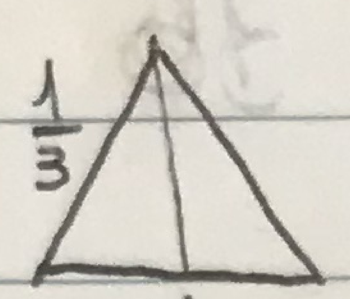
Step 2 • Find the factor λ.   increasing by "$\odot$" $\lambda = +\odot$
decreasing by "number" $\lambda = -$number

Step 3 • Match a so that the first terms (first 2 or 3) correspond.

Week 13

III 1/

area of the triangle at stage 1



$\frac{1}{3}$; $\frac{4}{3}$

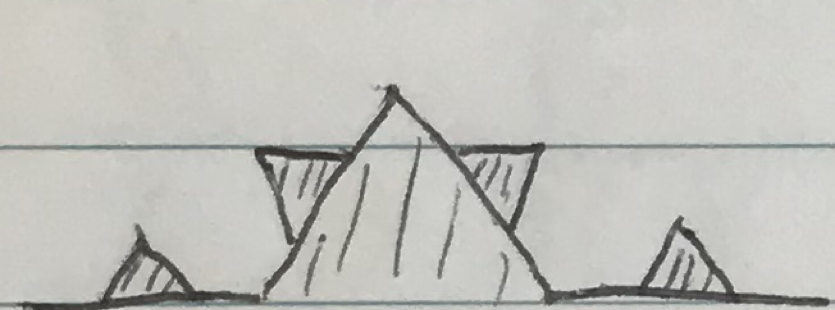
$\sqrt{\left(\frac{1}{3}\right)^2 - \frac{1}{6^2}} = \frac{\sqrt{3}}{6}$

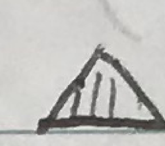
$\frac{1}{6}$

Area $\rightsquigarrow \frac{1}{3} \times \frac{\sqrt{3}}{6} \times \frac{1}{2}$

$A_1 = \frac{\sqrt{3}}{36}$

Triangle at stage 2



Area of 

Area of triangle at stage 2 = A_2

$A_2 = A_1 \times \left(\frac{1}{3}\right)^2$

$A_2 = \frac{A_1}{9}$

Area of the small triangle at stage n $\quad A_n = A_1 \times \left(\frac{1}{9}\right)^{n-1}$

T_n total area of the snowflake at stage n.

$T_0 = 0 \quad T_1 = A_1 = \frac{\sqrt{3}}{36}$

4 small triangles

$T_2 = \cancel{T_1} + \frac{\sqrt{3}}{36} + \frac{4}{9} \times \frac{\sqrt{3}}{36}$

$T_3 = \frac{\sqrt{3}}{36} + \frac{4}{9}\frac{\sqrt{3}}{36} + \frac{4 \times 4}{9^2}\frac{\sqrt{3}}{36}$

$$T_4 = \frac{\sqrt{3}}{36} + \frac{4}{9}\frac{\sqrt{3}}{36} + \frac{4\times4\times4}{9\times9\times9}\frac{\sqrt{3}}{36} + \frac{4^3}{9^3}\frac{\sqrt{3}}{36}$$

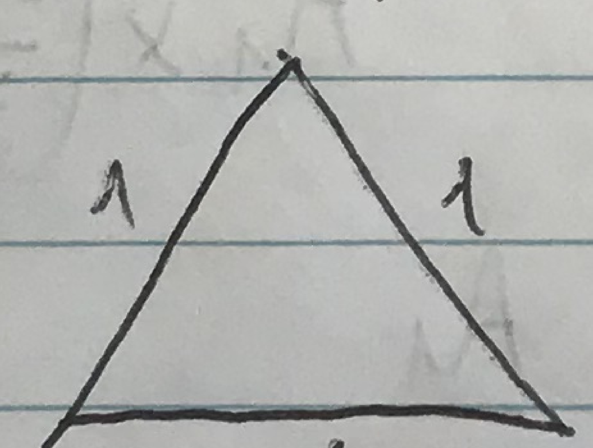
$\vdots$

$$T_n = \frac{\sqrt{3}}{36} + \frac{4}{9}\frac{\sqrt{3}}{36} + \cdots + \frac{4^{n-1}}{9^{n-1}}\frac{\sqrt{3}}{36}$$

$$= \frac{\frac{\sqrt{3}}{36} - \frac{4^n}{9^n}\frac{\sqrt{3}}{36}}{1-\left(\frac{4}{9}\right)},$$

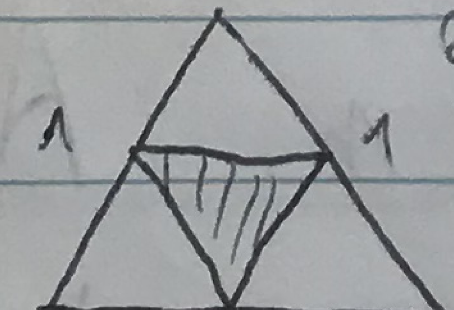
Sierpinski Gasket.

Take an equilateral triangle

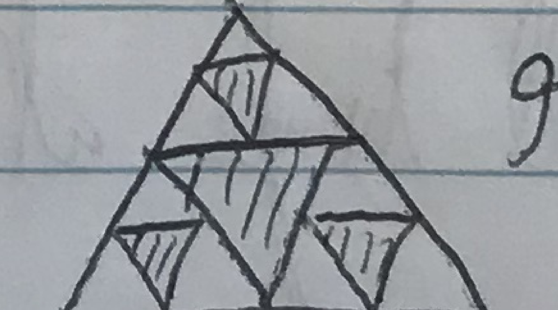


1 region

remove equilateral triangle



3 region



9 regions.

What is the area of the Sierpinski Gasket ?

Area of the first triangle $\left(\frac{\sqrt{3}}{2} \times \frac{1}{1}\right) \times \frac{1}{2} = \frac{\sqrt{3}}{4}$.

Area of each small triangle at stage 1. $\frac{\sqrt{3}}{16}$.

" " 2 $\frac{\sqrt{3}}{\cancel{32}\ 4\times4\times4}$

Week 13 III 2/

all stage n $\frac{\sqrt{3}}{4^{n+1}}$

Area of sierpinski at stage 1 $\quad \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{16}$

stage 2 $\quad \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{16} - 3 \times \frac{\sqrt{3}}{4^3}$

stage 3 $\quad \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{16} - \frac{3\sqrt{3}}{4^3} - 3^2 \frac{\sqrt{3}}{4^4}$

⋮

stage n $\quad \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{16} - \frac{3\sqrt{3}}{4^3} - \frac{3^2\sqrt{3}}{4^4} - \dots - 3^{n-1} \frac{\sqrt{3}}{4^{n+1}}$

$$\cancel{\frac{\frac{\sqrt{3}}{4} - \left[-\frac{3^n}{4^{n+2}}\sqrt{3}\right]}{1 -}}$$

$$\frac{\sqrt{3}}{4} - \left[\frac{\sqrt{3}}{16} + \frac{3}{4}\frac{\sqrt{3}}{16} + \dots + \frac{3^{n-1}\sqrt{3}}{4^{n+1}}\right]$$

$$\frac{\sqrt{3}}{4} - \frac{\frac{\sqrt{3}}{16} - \frac{3^n}{4^{n+2}}\sqrt{3}}{1 - \frac{3}{4}} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} - (\text{small})$$

$$= \frac{3^n}{4^{n+1}}\sqrt{3}.$$