

question = MIDTERM.
Double dice roll

Proba III
1/

Take a dice such that the probability
to obtain 1 is $\frac{1}{12} = p_1$

$$2 \text{ is } \frac{3}{12} = p_2$$

$$3 \text{ is } \frac{3}{12} = p_3$$

$$4 \text{ is } \frac{1}{6} = p_4$$

$$5 \text{ is } \frac{1}{6} = p_5$$

$$6 \text{ is } \frac{1}{12} = p_6$$

What is the probability
that I obtain the sum is ≥ 10 .
We suppose that the two throws are independent.

* S ? an element of S is a pair of numbers between 1 and 6.

* $\# S = 36$

* $P(\{(i, j)\}) = p_i \times p_j$

$A = \text{sum} \geq 10$

$P(A) = P(\{(6, 4), (5, 5), (4, 6), (6, 5), (5, 6), (6, 6)\})$

$= \frac{1}{12} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{12} \times \frac{1}{6} + \frac{1}{12} \times \frac{1}{6} +$

$\frac{1}{12} \times \frac{1}{12}$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{4} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{12} \times \frac{1}{12}$

$= \frac{1}{6} \left[\frac{1}{4} + \frac{1}{6} + \frac{1}{12 \times 2} \right]$

$= \frac{1}{6} \times \frac{101}{24}$

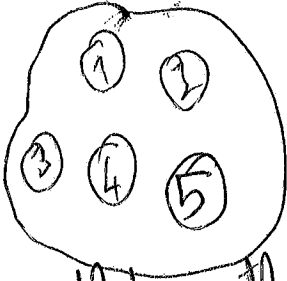
$$\#S = \frac{5 \times 4}{2}$$

Number of 2 combinations
of $\{1, \dots, 5\}$.

$$P(A) = \frac{2}{5 \times 4}$$
$$P(1 \text{ and } 2)$$

Fact: what is the number of unordered subset
of 3 elements of a set of 5 elements.
 $\frac{5 \times 4 \times 3}{3!}$ (# ordered list of 3 elts)
 \rightarrow (# permutations of 3 elts).
of unordered subset
of h elements of a set of n elements
 $\frac{n(n-1)\dots(n-h+1)}{h!}$

Multiple selection



Take 2 balls from the bag at once.

What is the probability that I take out 1 and 2?

Ordered method

$S =$ element are pairs of ~~number~~ distinct numbers from 1 to 5.

$\#S = 5 \times 4$

$A =$ take out 1 and 2.

$A = \{(1, 2), (2, 1)\}$

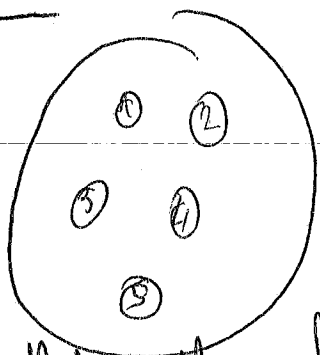
$IP(A) = \frac{2}{5 \times 4} = \frac{1}{10}$

Non ordered method.

$S?$ - an element of S is a ^{collection} (subset) containing 2 ~~elemen~~ 2 elts of $\{1, 2, \dots, 5\}$.

ex: $\{1, 5\} = \{5, 1\}$,

Same



take 3 balls out.

What is the probability that I take 1, 2, 3 out?

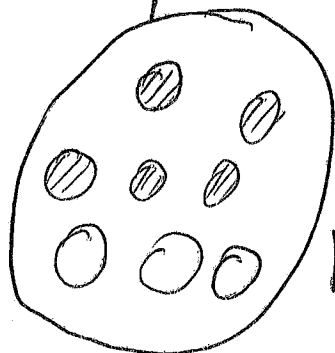
$S =$ unordered list of 3 elements (distinct).

ex: $\{1, 2, 3\}$.

$$\#S = \frac{5 \times 4 \times 3}{3!} = 10$$

$$P(\text{take } 123) = \frac{1}{10}$$

What if



5 3

Take out 3 balls. (at once).

Probability that I take 2 black 1 white.

$S =$ selection of 3 balls among 8.

$$\#S = \frac{8 \times 7 \times 6}{3!} = 56$$

~~#~~ $A =$ take 2 black and 1 white.

$$\#A = 3 \times \frac{5 \times 4}{2} = 30$$

↳ choose white ball

$$P(A) = \frac{30}{56}$$

• Cards selection:

54 cards deck.

Take out 2 cards
what is the probability that they have the same value?

• Independence

Take 1 card, shuffle, take another card.
put back
 $IP(\text{twice the same card})$.

Def: Two events are independent if $IP(E \text{ and } F) = IP(E) \times IP(F)$.

Ex: double coin toss (non balanced).

E : first is head

F : second is head.

$$IP(E) \times IP(F) = IP(\{HH\}) = \frac{1}{3} \times \frac{1}{3}$$

non indep: E = same result
 F = first is head

$E \text{ and } F = \{HH\}$

$$IP(E \text{ and } F) = \frac{1}{9}$$

$$IP(F) = \frac{1}{3}$$

$$IP(E) = \frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{5}{9}$$