

Probability

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Previously, we studied graphs, a beautiful abstract object in math that has many fruitful applications, inside math but also in computer science.

Today, I am going to start another cycle: Probability theory.

- What is probability? What is it about?
- When was it invented and by who?
- Why was it invented?

Pascal (1623-1662)

Pascal invented probability because he wanted to win more in card games.

In terms of the history of math, it is a young field compared to geometry, algebra. It took time for probability and statistics to be considered as mathematics, because of how it was born.

Modern probability was developed by Kolmogorov (1903-1987).
Foundations of the theory of probability (1933).
where he invented the modern language.

BABY EXAMPLES

• If I toss a coin, why is there 50% chance to fall into 'a head'?

Pascal's idea: count all the possible different outcomes of the coin toss.

Decree that all these outcomes have the same probability to happen.

Compute
Kolmogorov's formalization (very abstract).

Put all the outcomes in a set S
called the sample space.

$$S = \left\{ \begin{array}{c} H \\ \uparrow \\ \text{head} \end{array}, \begin{array}{c} T \\ \uparrow \\ \text{tail} \end{array} \right\} \quad \# \text{outcomes} = \# S$$

$$A = \{H\}$$

$P(A)$ = probability that our toss gives head. = $\frac{1}{2}$

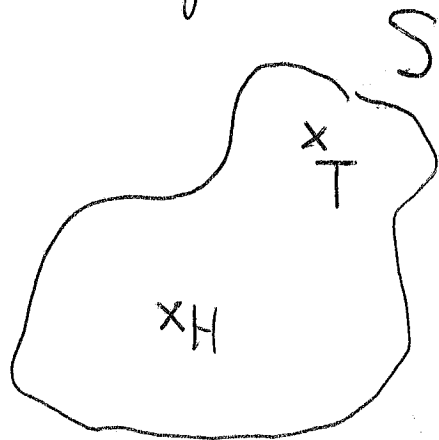
$P(\{T\})$ = " tails = $\frac{1}{2}$

$P(\{H, T\})$ = probability that our toss gives head or tails.
= 1.

You have to remember this baby example.

Rh: Sounds very dumb - but this example is very important and usually the hardest thing is to describe the sample space S .

Usually (all the time), people often forget about it.



BABY II

I toss a perfectly balanced coin twice
what is the probability that I get
two heads?

that I get twice the same result?

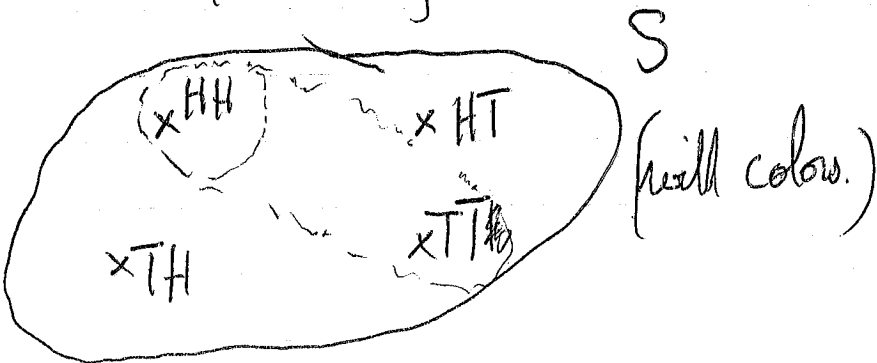
Tip: how to determine the sample space is imagine all the things
that can happen if you actually do it.
Then try to route it down

~~$S = \{H, T\}$~~ $S = \{ \overset{\substack{\downarrow \text{2nd toss.} \\ \text{HH, TT, HT, TH}}}{\uparrow \text{1st toss.}} \}$

$$P(\{HH\}) = \frac{1}{4}$$

$$P(\text{twice the same result}) = \frac{2}{4} = \frac{1}{2}$$

$\{HH, TT\}$



Baby II'

loss a non-balanced coin?
with probability $\frac{1}{3}$ heads
 $\frac{2}{3}$ tails

Baby II'' 4 loss, 5 loss? 6 losses?
n losses.

Baby III double dice roll.

Def. Events subset of S (formal definition).

ex: english sentence = "get twice the same result"
"get two heads"
Mathematically : {HH, TT}
{HH}.

