MAT 200 **Name:** ____________________

Instructor: Alena Erchenko

Fall 2019

Midterm 1

10/02/2019

Time Limit: 53 Minutes

This exam contains 7 pages (including this cover page) and 5 problems. Write your name on the top of this page.

YOU MAY *NOT* USE YOUR BOOKS, NOTES, OR ANY CALCULATOR ON THIS EXAM. PLEASE TURN OFF AND PUT AWAY YOUR CELL PHONE.

You are required to show your work on each problem on this exam. **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct explanations might still receive partial credit.

If you need more space, use the back of the pages or extra page at the end of the exam; clearly indicate when you have done this. Do not rip off pages from the exam.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (20 points) (a) (5 points) State the negation of the statement "For all real numbers x there exists a natural number q such that $xq = 36$".

(b) (5 points) State the negation of the statement "$\exists x \in \mathbb{Z}$, such that $\forall q \in \mathbb{R}, x + q > 0$ or $xq = 0$".

(c) (5 points) State the contrapositive of the statement "If $p < q$, then $p = 0$ or $q = 1$".

(d) (5 points) State the converse of the statement "If $p < q$, then $p = 0$ or $q = 1$".

2. (20 points) Below A, B, and C denote arbitrary sets. Are the following statements True or False? Either provide a counterexample or prove that the statement is true.

   (a) (5 points) If $A \setminus B = \emptyset$, then $A = B$.

   (b) (5 points) If $A \subset (B \cup C)$ and A has at least 2 elements, then both B and C must have at least one element.

   (c) (10 points) $C \setminus (A \setminus B) = (C \setminus A) \cup (C \cap B)$.

3. (20 points) Prove that for all integer numbers x, we have $x(x+1)$ is even.

   [You may suppose that an integer n is odd if and only if $n = 2q + 1$ for some integer q.]

4. (20 points) Show that there are no natural numbers x such that $x^3 - x^2 + x + 1 = 0$.

   [You can use the fact that the only natural number that divides 1 is 1.]

5. (20 points) Given a sequence of numbers $a(2), a(3), \ldots$, the number $\prod_{i=2}^{n} a(i)$ is defined inductively by

   1. $\prod_{i=2}^{2} a(i) = a(2)$, and
   2. $\prod_{i=2}^{k+1} a(i) = \left(\prod_{i=2}^{k} a(i)\right) a(k+1)$ for $k \geq 2$.

   Prove that for all integers $n \geq 2$ we have

$$\prod_{i=2}^{n} \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}.$$

(EXTRA PAGE 1)