MAT 200 Spring 2019 Midterm

Name: **ID:**

There are four problems with indicated values.

Show your work. Continue writing on the back of the pages if needed.

Do not tear-off any page(s).

No calculators, no cellphones, etc.

1	26	
2	24	
3	25	
4	25	
Total	100	

Name: ______________________________ Problem 1: ________/26

Problem 1 (26 points) In each of the following statements, circle T if it is true and F if it is false. There is no need to show your work in this problem.

T F (1) The statement 'For real numbers a, if a is non-positive then $f(a) \neq 0$.' is the negation of the statement 'For real numbers a, if $f(a) = 0$ then a is negative.'

T F (2) The statement 'For some non-positive real number a, $f(a) = 0$.' is the negation of the statement 'For real numbers a, if $f(a) = 0$ then a is negative.'

T F (3) The statements '(not P) $\Longrightarrow$ Q' and '(not Q) $\Longrightarrow$ P' are equivalent.

T F (4) The statements 'P or Q' and '(not P) $\Longrightarrow$ Q' are equivalent.

T F (5) Let A, B, and C be sets. Then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

T F (6) Let A, B, C, and D be sets. Then $(A \cap B) \cup (C \cap D) = (A \cup C) \cap (A \cup D) \cap (B \cup C) \cap (B \cup D)$.

T F (7) There is no such function $f : [0, 2] \to \mathbb{Z}$ satisfying the following property: for all $\epsilon > 0$ there exists $\delta > 0$ such that for all $x \in (1 - \delta, 1 + \delta) \cap [0, 2]$, $|f(x) - f(1)| < \epsilon$.

T F (8) For all sets X and Y, $X \times Y = Y \times X$.

T F (9) There exist sets X and Y such that $X \times Y = Y \times X$.

T F (10) $\{f : \mathbb{R} \to X \,|\, X \text{ is a set and } f(x) = f(-x) \text{ for each } x \in (0, +\infty)\}$ is NOT a set.

T F (11) $\{f : \mathbb{R} \to \mathbb{R} \,|\, f(x) = f(-x) \text{ for each } x \in (0, +\infty)\}$ is a set.

T F (12) $\{f : \mathbb{R} \to \mathbb{R} \,|\, f(x) \neq f(x) \text{ for each } x \in \mathbb{R}\}$ is NOT a set.

T F (13) Given $f : X \to Y$ and $g : Y \to Z$. Then $f \circ g$ is a function whose domain is X.

Name: ________________________________ Problem 2: ________/24

Problem 2 (24 points) Prove that for all $x, y \in \mathbb{R}$, $x < y \implies x^3 < y^3$.

Name: ______________________________ Problem 3: ________/25

Problem 3 (25 points) Prove for all integer n, n^2 is even $\implies$ n is even.

Name: ______________________________ Problem 4: ________/25

Problem 4 (25 points) Prove by induction on n that

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1},$$

for all positive integers n.