MAT 200 Midterm 2
Sample Questions

1. Recall that $\mathrm{Fun}(X, Y)$ is the set of all functions from X to Y, and $\mathbb{N}_n = \{1, 2, \ldots, n\}$, and $\mathcal{P}(X)$ is the set of all subsets of X.

(a) Suppose Y has m elements. How many elements does the set $\mathrm{Fun}(\mathbb{N}_3, Y)$ have? (No justification needed, just write your answer).

(b) Give an example of a function with domain $\mathrm{Fun}(\mathbb{N}_3, Y)$ and codomain $\mathcal{P}(Y)$.

(c) Suppose Y has m elements. How many elements does the set $\mathrm{Fun}(\mathrm{Fun}(\mathbb{N}_3, Y), \mathcal{P}(Y))$ have? (No justification needed, just write your answer).

(d) Suppose Y has m elements and $m \geq 2$. How many injective functions are there from $\mathrm{Fun}(\mathbb{N}_3, Y)$ to $\mathcal{P}(Y)$? Justify your answer.

(e) Consider the following set. $\mathrm{Fun}(X, Y) \cup (\mathbb{N}_3 \times Y) \cup (\{3, 4, 5\} \times Y)$ How many elements are in this set? Your answer should be in terms of m and n. (No justification needed).

2. Let X be a set. Find a bijection between the sets 2^X and $\mathrm{Fun}(X, \{2, 6\})$. Prove that it is a bijection.

3. Let $A = \{x \in \mathbb{R} | x \geq 0\}$ and $B = \{y \in \mathbb{R} | y \geq 5\}$. Consider the function $f \colon A \to B$ defined by $f(x) = 9x^2 + 5$ for all $x \in A$.

(a) Show that f is injective using the definition of an injective function.

(b) Show that f is surjective using the definition of a surjective function.

4. Let $f \colon X \to X$ be an injection. Prove that if $f \circ f = f$ then $f = \mathrm{id}_X$.

5. Let $g \colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $g(x, y) = x + y$. Is g surjective? Is g injective?

6. Consider the function $f \colon \mathbb{Z} \to \mathbb{Q}$ defined by $f(n) = n/(|n| + 1)$.

(a) What is the domain of f? What is its image?

(b) Prove or disprove: f is surjective.

(c) Prove or disprove: f is injective.

(d) Draw (part of) the graph of f.

7. Let $f\colon X \to Y$ and $g\colon Y \to Z$ be functions. Show that if $g \circ f\colon X \to Z$ is an injection, then f is an injection.

8. (Pr. III.1) Out of 182 students who are taking three first year core Mathematics modulues (Reasoning, Algebra and Calculus), 129 like Reasoning, 129 like Algebra, 129 like Calculus, 85 like Reasoning and Algebra, 89 like Reasoning and Calculus, 86 like Algebra and Calculus, and 54 like all three modulues. How many of the students like none of the core modules?