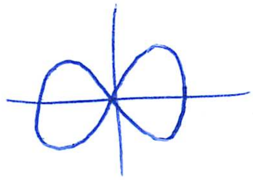
1. $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 3 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ -1 & 6 \end{bmatrix}$ $\left(\det \begin{bmatrix} 11 & -1 \\ -1 & 6 \end{bmatrix} \right)^{1/2} = \sqrt{66-1} = \boxed{\sqrt{65}}$

2. $A = \begin{bmatrix} 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \end{bmatrix}$

Let a, b denote missing entries.
By orthogonality, $\begin{cases} \frac{a}{\sqrt{3}} + \frac{b}{\sqrt{3}} + \frac{1}{\sqrt{6}} = 0 \\ \frac{-2a}{\sqrt{6}} + \frac{b}{\sqrt{6}} + \frac{1}{\sqrt{12}} = 0 \end{cases}$

$\Longrightarrow \begin{cases} a + b + \frac{1}{\sqrt{2}} = 0 \\ -2a + b + \frac{1}{\sqrt{2}} = 0 \end{cases} \Longrightarrow a = 0, \quad b = -1/\sqrt{2}.$

3. Y is not a manifold. Y is a "bowtie" ,

so the origin is not a manifold point.

Alternatively, $\alpha((-\varepsilon, \varepsilon))$ is not open in Y for $0 < \varepsilon < \pi$.

4. Compute $D\alpha = \begin{bmatrix} -\cos(\varphi)\sin(\theta) & -\sin(\varphi)(2+\cos(\theta)) \\ -\sin(\varphi)\sin(\theta) & \cos(\varphi)(2+\cos(\theta)) \\ \cos(\theta) & 0 \end{bmatrix}$

$(D\alpha)^T \cdot D\alpha = \begin{bmatrix} 1 & 0 \\ 0 & (2+\cos(\theta))^2 \end{bmatrix}$ $\det(D\alpha^T \cdot D\alpha)^{1/2} = 2 + \cos(\theta)$

$\int_0^{2\pi} \int_0^{2\pi} (2 + \cos(\theta))\, d\theta\, d\varphi = \boxed{8\pi^2}$

5. $f = 3\phi_1 \otimes \phi_2 \otimes \phi_3 - \phi_3 \otimes \phi_3 \otimes \phi_4$

6. $\phi_1 \otimes \phi_4 \otimes \phi_3 (x,y,z) = 1 \cdot 1 \cdot 0 = \boxed{0}$

$$\phi_1 \wedge \phi_4 \wedge \phi_3 (x,y,z) = \det \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = 1 \cdot (0-1) = \boxed{-1}$$

7. (a) Yes
(b) No
(c) Yes

8. $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$. This is the number of ascending k-tuples from a set of n elements.