Midterm 1 Key

1. (a) $\vec{u}\cdot\vec{v} = 2-6+3 = -1$

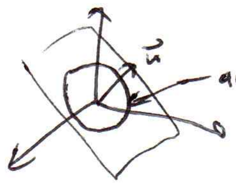
$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & -3 & 1 \end{vmatrix} = (2+9)\vec{i} - (1-6)\vec{j} + (-3-4)\vec{k}$
$= \boxed{11\vec{i} + 5\vec{j} - 7\vec{k}}$

(b) $\boxed{\text{obtuse}}$ since $\vec{u}\cdot\vec{v} < 0$

(c) area $= \dfrac{\|\vec{u}\times\vec{v}\|}{2} = \dfrac{\sqrt{121+25+49}}{2} = \boxed{\dfrac{\sqrt{195}}{2}}$

(d) $\vec{w} = \dfrac{\vec{u}}{\|\vec{u}\|} = \dfrac{\langle 1,2,3\rangle}{\sqrt{1+4+9}} = \boxed{\left\langle \dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}} \right\rangle}$

(e) $\boxed{\text{infinitely many}}$



any vector on this circle

2. (a) $\vec{PQ} = \langle 2,2,1\rangle$ $\quad \vec{PR} = \langle -3,1,1\rangle$

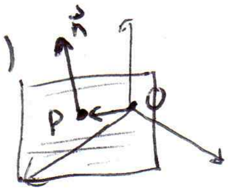
$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 1 \\ -3 & 1 & 1 \end{vmatrix} = (2-1)\vec{i} - (2+3)\vec{j} + (2+6)\vec{k} = \vec{i} - 5\vec{j} + 8\vec{k}$

$1(x-1) - 5(y+1) + 8(z-0) = 0$

$x - 1 - 5y - 5 + 8z = 0 \qquad \boxed{x - 5y + 8z = 6}$

(b) $\boxed{x = 1+t, \quad y = -1-5t, \quad z = 8t}$

(c)



$d = \dfrac{|\vec{OP}\cdot\vec{n}|}{\|\vec{n}\|} = \dfrac{\langle 1,-1,0\rangle \cdot \langle 1,-5,8\rangle}{\sqrt{1+25+64}} = \dfrac{1+5}{\sqrt{90}} = \dfrac{6}{3\sqrt{10}} = \boxed{\dfrac{2}{\sqrt{10}}}$

3. (a) $\vec{r}(t) = \displaystyle\int_0^t \vec{r}'(u)\,du = \langle -\cos(t), \sin(t), t\rangle - \vec{r}_0$

$\vec{r}(0) = \langle -1, 0, 0\rangle - \vec{r}_0 = \langle 0,0,0\rangle \Longrightarrow \vec{r}_0 = \langle -1,0,0\rangle$

$\boxed{\vec{r}(t) = \langle 1-\cos(t), \sin(t), t\rangle}$

(b) $\vec{T}(t) = \dfrac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \dfrac{\langle \sin(t), \cos(t), 1\rangle}{\sqrt{\sin^2(t)+\cos^2(t)+1}} = \boxed{\left\langle \dfrac{\sin(t)}{\sqrt{2}}, \dfrac{\cos(t)}{\sqrt{2}}, \dfrac{1}{\sqrt{2}} \right\rangle}$

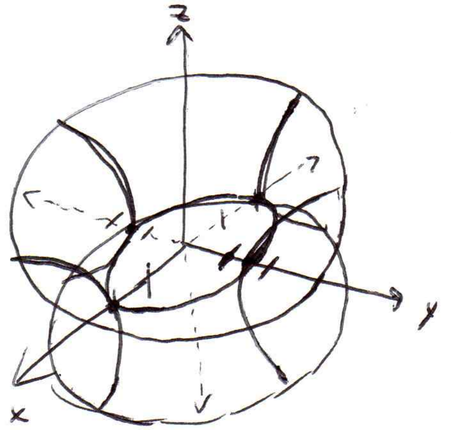
(c) $\vec{N}(t) = \dfrac{\vec{T}'(t)}{\|\vec{T}'(t)\|} = \boxed{\langle \cos(t), -\sin(t), 0\rangle}$

$\vec{T}'(t) = \left\langle \dfrac{\cos(t)}{\sqrt{2}}, \dfrac{-\sin(t)}{\sqrt{2}}, 0 \right\rangle$

$\|\vec{T}'(t)\| = \sqrt{\dfrac{\cos^2(t)}{2} + \dfrac{\sin^2(t)}{2}} = \dfrac{1}{\sqrt{2}}$

(d) $K = \frac{\|\vec{T}'(\frac{\pi}{2})\|}{\|\vec{r}'(\frac{\pi}{2})\|} = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$

4. (a)



(b) hyperboloid of one sheet

(c) $f(x,y) = \sqrt{x^2+2y^2-4}$

$f_x(x,y) = \frac{2x}{2\sqrt{x^2+2y^2-4}} = \frac{x}{\sqrt{x^2+2y^2-4}}$ $\quad f_y(x,y) = \frac{4y}{2\sqrt{x^2+2y^2-4}} = \frac{2y}{\sqrt{x^2+2y^2-4}}$

$f_x(2,1) = \frac{4}{2\sqrt{4+2-4}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ $\quad f_y(2,1) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$$z = \sqrt{2}(x-2) + \sqrt{2}(y-1) + \sqrt{2}$$

5. (a) $\boxed{\text{iii}}$ (b)

(c) $\nabla f(x,y) = \langle 2xe^{x^2-y^2}, -2ye^{x^2-y^2} \rangle$ $\quad \nabla f(1/2, 1/2) = \langle 2/2\, e^0, -2/2\, e^0 \rangle = \langle 1, -1 \rangle$

$\nabla f(1/2,1/2) \cdot \langle 2/\sqrt{5}, 1/\sqrt{5} \rangle = \langle 1, -1 \rangle \cdot \langle 2/\sqrt{5}, 1/\sqrt{5} \rangle = \boxed{1/\sqrt{5}}$

(d) $\frac{dz}{dt} = \frac{dz}{dx}\frac{dx}{dt} + \frac{dz}{dy}\frac{dy}{dt} = 2(\pi-1)^3 e^{(\pi-1)^6-1}\, 3(\pi-1)^2 + 2e^{(\pi-1)^6-1}(-1)$

$x(\pi) = (\pi-1)^3$

$y(\pi) = -1 + 0 = -1$

$\frac{dx}{dt}(\pi) = 3(\pi-1)^2$

$= (6(\pi-1)^5 - 2)\, e^{(\pi-1)^6-1}$