Exploring the graphing calculator

Problems 1 and 2 are based on the problems from the book "Inside your graphing calculator" by Gerald Rising

1. The number 3816547290 has the property that, for some positive integers N, when N digits are removed from the right, the result is divisible by $10 - N$. Find for the values of N for which this property holds.

2. The square if 428 is 183184: a number whose first three digits and last three digits form numbers that differ by one. There are only three other three digit numbers whose squares have that property. Find them.

3. Choose a three digit number in your calculator. Multiply it by 7, multiply it by 11 and multiply it by thirteen. The result is "mathmagic": your original number should reappear twice. Can you explain (mathematically) what happened? State a conjecture and prove it.

4. (with your calculator) Choose a positive number. Compute the square root. Compute the square root of the result again and again... What do you observe? Explain it mathematically. Does your explanation apply to every positive number?

5. Describe an strategy so player B wins the following game.

   (a) Player A enters a three digit number smaller than 900 in the calculator

   (b) Player B must reduce the given number to zero in at most 5 steps, using only the four basic operations and single digit numbers. Player B is only allowed to use exact divisors.

   (c) Here is an example of the game:
      (i) Player A enters the number 678.
      (ii) Player B divides by 6, obtaining 113.
      (iii) Player B subtracts 5, obtaining 108.
      (iv) Player B divides by 6, obtaining 18.
      (v) Player B divides by 6 obtaining 3.
      (vi) Player B subtracts 3, obtaining 0.