MAT 211 H.W. 3 Solution

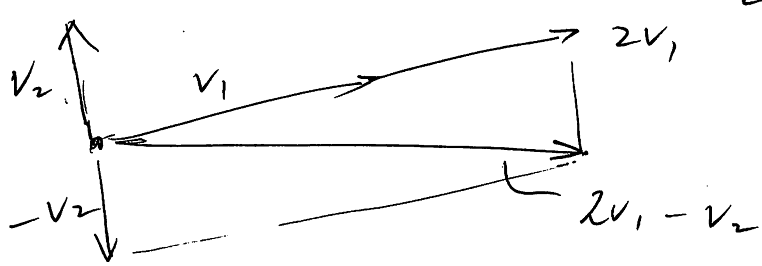
2.1.38) Let $v_1 = \begin{pmatrix} v_{11} \\ v_{21} \end{pmatrix}$ $v_2 = \begin{pmatrix} v_{12} \\ v_{22} \end{pmatrix}$

then $A = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix}$

$$A\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2v_{11} \\ 2v_{21} \end{pmatrix} - \begin{pmatrix} v_{12} \\ v_{22} \end{pmatrix}$$

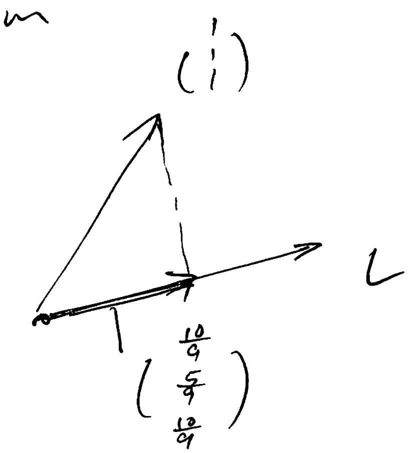
$$= 2v_1 - v_2$$



2.2.6) $$\text{proj}_L\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right) = \underbrace{\frac{\left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\rangle}}_{\text{scalar}} \underbrace{\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}}_{\text{Vector}}$$

$$= \frac{5}{9}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{10}{9} \\ \frac{5}{9} \\ \frac{10}{9} \end{pmatrix}$$

Geometric Interpretation



2.2.30) * This question has infinitely many solutions, but we'll follow the book.

Let x be any vector on the plane, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

then if we project x on the line parallel to $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, we get

$$\text{proj}_L(x) = \frac{\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \rangle}{\langle \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \rangle} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{x_1 + 2x_2}{5} \\ \frac{2}{5}(x_1 + 2x_2) \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$\therefore$ One such A is $\begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$

However, in general, any non-zero $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ that satisfies $a_{21} = 2a_{11}$, $a_{22} = a_{12}$ will do

To see this. $\begin{pmatrix} a_{11} & a_{12} \\ 2a_{11} & 2a_{12} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ 2a_{11}x_1 + 2a_{12}x_2 \end{pmatrix}$

$$= (a_{11}x_1 + a_{12}x_2) \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

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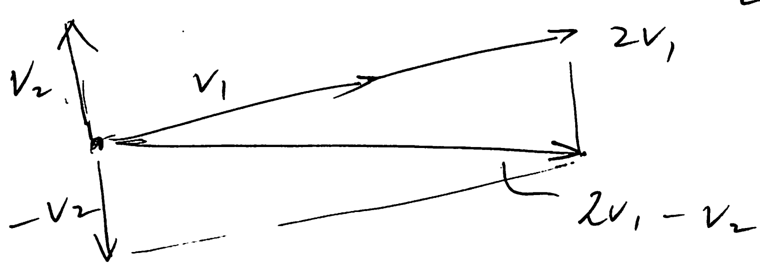

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