Math 312/ AMS 351 (Fall '17)
Sample Questions for Midterm 1

1. Find the gcd of 6, 14, 21 and express it in the form $6r + 14s + 21t$ for some integers r, s, and t.

2. Find a such that for all positive integers n, it holds

$$1 + 4 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{a}$$

(Note: the question consists of two parts: find the correct value of a, and secondly you need to prove that the formula holds for all a)

3.   i) Show that, for every positive integer n, $n^5 - n$ is divisible by 5.

  ii) Show that, for every positive integer n, $3^{2n} - 1$ is divisible by 8.

4. Show that there exist infinitely many primes of the form $4k + 3$.

5. Show that the equation $7x^3 - 6x^2 + 2x - 1 = 0$ has no integer solutions.

6. Prove that a number n is divisible by 11 iff the alternating sum of its digits is divisible by 11 (e.g. $n = 1234$ has the alternating sum of digits equal to $1 - 2 + 3 - 4 = -2$ and thus it is not divisible by 11, while 132 is divisible by 11).

7. Solve the following equations:

   i) $15x = 5 \bmod 18$

  ii) $15x = 5 \bmod 17$

 iii) $3x = 1 \bmod 5$ and $2x = 6 \bmod 8$

8. Show that no integer of the form $8n + 7$ can be written as the sum of 3 squares.

9. Show that, for every integer n, $n^{13} - n$ is divisible by $2, 3, 5, 7$.

10. Find the last 2 digits of 2^{123}.