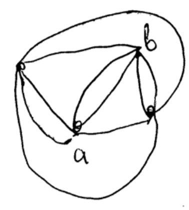
MAT310 Fall 2012
Practice Midterm II

The actual midterm will consist of six problems. You will be allowed to use calculators. Sections covered:2.7-4.2(inclusive) with the exception of economics applications.

Problem 1 1. Find the number of path of length four connecting points *a* and *b* in the graph



2. Find the number of loops of length four in the same graph.

Problem 2

1. Find all λ such that the equation $y'' = \lambda y$ has a solution $y(t)$ with $y(0) = y(1) = 0$

2. Find the general solution of

$$y^{(4)} + 2\,y^{(3)} + 10\,y^{(2)} + 18\,y + 9 = 0$$

Problem 3 1. Compute the rank the matrix

$$\begin{bmatrix} 2 & 6 & 1 & 0 \\ 3 & 3 & 0 & 1 \\ -2 & -2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

.

2. Complete the set of two vectors $(2, 3, -2, 1)$ and $(6, 3, -2, 1)$ to a basis of $\mathbb{R}^4$.

3. Which of the monomials $p_i(x) = x^i, i = 0, \ldots, 3$ is a linear combination of $q_1(x) = 2 + 3x - 2x^2 + x^3$ and $q_1(x) = 6 + 3x - 2x^2 + x^3$

Problem 4 1. Compute the matrix inverse of

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ -1 & 1 & -3 & 1 \\ 0 & 2 & 3 & 0 \\ -5 & 1 & -15 & 0 \end{bmatrix}$$

2. Use A^{-1} to solve

$$x_1 + 2\,x_2 = 5$$

$$-3\,x_1 + x_2 = 6$$

Problem 5 Compute the determinant of

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 0 & 9 & 10 \\ 0 & 0 & 11 & 12 \end{bmatrix}$$

Problem 6 Determine the values of a for which the system .

$$x_1 + 2\,x_2 - x_3 + x_4 = 5$$

$$x_1 + x_2 - 2\,x_3 + 3\,x_4 = 6$$

$$-2\,x_1 + 6\,x_3 - 10\,x_4 = a$$

is consistent. Find the basis in the space of solution for such a.

Problem 7 1. What happens to the determinant of $n \times n$ matrix if you swap the first k rows with the remaining $n-k$ rows.

2. A matrix $A \in Mat_{10,10}$ is a product of $B \in Mat_{10,5}$ and $C \in Mat_{5,10}$. Why $\det A = 0$?

Problem 8 Label the following statements as being true or false or being incorrectly stated.

1. A square matrix with positive entries has a nonnegative determinant.
2. The set of vectors $(u_1, \ldots, u_k)$ is a basis in a k-dimensional subspace $V \subset \mathbb{F}^n$. Then the determinant of a matrix, whose columns are $u_1, \ldots, u_k$ is nonzero.
3. Determinant of an elementary matrix is equal to ± 1.
4. Determinant of an lower triangular matrix is is a product of diagonal terms.
5. If a square matrix contains two identical entries then its determinant is zero.
6. Any *rank n* matrix $A \in Mat_{n,n}$ is invertible.
7. If a matrix $A \in Mat_{n,n}$ is invertible, then it contains no nonzero entries.

Problem 9

1. The short arrow of a clock represents the first vector of a basis in $\mathbb{R}^2$, the long represents the second vector. What is the orientation of this basis at 3:10, 6:05 and 7:10. What can you say about the time 12:00?
2. What is the orientation of (u, v), where $u = (1, -5)$, $v = (-2, 3)$? What is the area of the parallelogram spanned by u and v?

Problem 10 Present

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

as a product of elementary matrices

Problem 11 Show that $rank(A + B) \leq rank(A) + rank(B)$ for $A, B \in Mat_{m,n}$