1.1

$\frac{dP}{dt} \geq 0$ at least for small t since the flu spreads.

$\frac{P(100-P)}{100} \geq 0$ since $0 \leq P(t) \leq 100$.

Therefore, $\boxed{k \geq 0.}$

1.2

$$\frac{100\, dP}{P(100-P)} = k\, dt$$

$$\frac{100}{P(100-P)} = \frac{A}{P} + \frac{B}{100-P} \implies A(100-P) + B \cdot P = 100$$

$$\implies A = B = 1$$

$$\left(\frac{1}{P} + \frac{1}{100-P}\right) dP = k\, dt$$

$$\ln P - \ln(100-P) = kt + C$$

$$\frac{P}{100-P} = Ae^{kt}$$

$$P = \frac{100\, Ae^{kt}}{1 + Ae^{kt}}$$

$$P(0) = 1 \implies 1 = \frac{100A}{1+A} \implies A = \frac{1}{99}$$

$$\boxed{P(t) = \frac{100e^{kt}}{99 + e^{kt}}.}$$

1.3

$$\lim_{t \to \infty} P(t) = \lim_{t \to \infty} \frac{k\, 100\, e^{kt}}{k e^{kt}} \quad \text{by L'Hospital's rule}$$

$$= \boxed{100.}$$

2.1 $\quad v' - \frac{1}{x}v = 2x \quad , \quad v(1) = 0$

Integrating factor: $\quad \rho(x) = e^{\int -\frac{1}{x}dx} = x^{-1}$

$$x^{-1}\left(v' - \frac{1}{x}v\right) = x^{-1}(2x)$$

$$x^{-1}v' - x^{-2}v = 2$$

$$(x^{-1}v)' = 2$$

$$x^{-1}v = 2x + A$$

$$v = 2x^2 + Ax$$

$v(1) = 0 \implies 0 = 2(1)^2 + A(1) \implies A = -2$

$\boxed{v = 2x^2 - 2x}$

2.2 $\quad (1+x)\frac{dy}{dx} = 4y \qquad y(0) = 1$

$$\frac{dy}{y} = \frac{4}{1+x} \qquad (\text{assuming } y \neq 0)$$

$$\ln|y| = 4\ln|1+x| + C$$

$$y = A(x+1)^4$$

$y(0) = 1 \implies 1 = A(0+1)^4 \implies A = 1$

$\boxed{y = (x+1)^4}$

2.3 $(xy')^2 + y^2 = 1, \quad y(1) = 0$

$$xy' = \pm\sqrt{1-y^2}$$

$$\frac{y'}{\sqrt{1-y^2}} = \pm \frac{1}{x}$$

$$(\arcsin y)' = \pm \frac{1}{x}$$

$$\arcsin y = \pm \ln|x| + C$$

$$y(1) = 0 \Rightarrow \quad 0 = \pm \ln 1 + C \quad \Rightarrow \quad C = 0$$

$$y = \pm \sin(\ln|x|)$$

2.4 $v' - \frac{v}{x} = xv^6, \quad v(0) = 1$

Let $w = \frac{v}{x}$. Then $v' = w + w'x$.

Then $w + w'x - w = x(wx)^6$

$$w' = w^6 x^6$$

$$w^{-6}\,dw = x^6\,dx$$

$$-\frac{1}{5}w^{-5} = \frac{1}{7}x^7 + C$$

$$w^{-5} = -\frac{5}{7}x^7 + B \qquad (\text{where } B = -5C)$$

$$v^{-5} = -\frac{5}{7}x^2 + Bx^{-5}$$

$$v = \left[-\frac{5}{7}x^2 + Bx^{-5}\right]^{-1/5}$$

There is no solution ~~corresponding~~ with initial value $v(0) = 1$.

3.1 $(4xy)\,dx + (6yx)\,dy = 0$

This differential equation is not exact since

$$\frac{\partial}{\partial y} 4xy = 4x \neq \frac{\partial}{\partial x} 6yx = 6y$$

However, dividing through by $6xy$ (assuming $y \neq 0$) gives an exact equation:

$$\frac{2}{3}\,dx + dy = 0$$

The solution has the form $F(x,y) = c$ where $\frac{\partial F}{\partial x} = \frac{2}{3}$ and $\frac{\partial F}{\partial y} = 1$.

$$\frac{\partial F}{\partial x} = \frac{2}{3} \Rightarrow F = \int \frac{2}{3}\,dx = \frac{2}{3}x + g(y)$$

$$\frac{\partial F}{\partial y} = 1 \Rightarrow g'(y) = 1 \Rightarrow g = y + \tilde{c}$$

Thus, $\boxed{\frac{2}{3}x + y = c}$, or $\boxed{y(x) = 0}$ in the case $y \equiv 0$.

3.2 $(xy+y^2)\,dx + x^2\,dy = 0$

This is not exact since

$$\frac{\partial}{\partial y}(xy+y^2) = x + 2y \quad , \qquad \text{and}$$

$$\frac{\partial}{\partial x} x^2 = 2x$$

are not equal.

3.3 However, it can be rewritten as

$$\frac{dy}{dx} = -\frac{xy+y^2}{x^2} = -\frac{y}{x} - \left(\frac{y}{x}\right)^2$$

Making the substitution $w = \frac{y}{x}$ we have

$$xw' + w = -w - w^2$$

$$\frac{w'}{w(2+w)} = -\frac{1}{x}$$

Using partial fractions we can write

$$\frac{1}{2}\left[\frac{1}{w} - \frac{1}{2+w}\right] w' = -\frac{1}{x}$$

$$\frac{1}{2}\left[\ln|w| - \ln|2+w|\right] = -\ln|x| + c$$

$$\ln\frac{w}{2+w} = -2\ln x + \tilde{c}$$

$$\frac{w}{2+w} = Ax^{-2}$$

Therefore $\quad w = \dfrac{2Ax^{-2}}{1-Ax^{-2}}$

Hence $\quad \boxed{y = \dfrac{2Ax^{-1}}{1-Ax^{-2}}}$