

**Problem 3.**

- Find critical points of the differential equation

$$y' = (y-1)(y+3) + c$$

- Find bifurcation points  $c$ .
- Find stable and unstable critical points
- Draw the bifurcation diagram

$$\begin{aligned} \bullet \quad y' = 0 &\Rightarrow y^2 + 2y - 3 + c = 0 \\ &\Rightarrow y^2 + 2y + 1 - 3 + c = 1 \\ &\Rightarrow (y+1)^2 = 4 - c \\ &\Rightarrow y = \boxed{-1 \pm \sqrt{4-c}} \quad \text{provided } c \leq 4. \end{aligned}$$

$$\bullet \quad 4 - c = 0 \Rightarrow \boxed{c = 4}$$

- $c = 4 \Rightarrow -1$  is only critical point. It is unstable
- $c < 4 \Rightarrow -1 \pm \sqrt{4-c}$  are critical points.

$$\begin{array}{c} \leftarrow \bullet \rightarrow \quad \leftarrow \bullet \rightarrow \\ -1 - \sqrt{4-c} \quad -1 + \sqrt{4-c} \end{array}$$

$-1 - \sqrt{4-c}$  is stable  
 $-1 + \sqrt{4-c}$  is unstable

