

3.3.8 $y'' - 6y' + 13y = 0$

$$r^2 - 6r + 13 = 0 \Rightarrow r = \frac{6 \pm \sqrt{36 - 52}}{2}$$

$$= 3 \pm 2i$$

$$y = e^{3x} (c_1 \cos(2x) + c_2 \sin(2x))$$

3.3.24 $2y^{(3)} - 3y'' - 2y' = 0$

$$y(0) = 1, y'(0) = -1, y''(0) = 3$$

$$2r^3 - 3r^2 - 2r = 0 \Rightarrow r(2r^2 - 3r - 2) = 0$$

$$\Rightarrow r(2r^2 - 4r + r - 2) = 0$$

$$\Rightarrow r(2r(r-2) + (r-2)) = 0$$

$$\Rightarrow r(2r+1)(r-2) = 0$$

$$\Rightarrow r = 0, -\frac{1}{2}, 2$$

$$y = c_1 + c_2 e^{-\frac{1}{2}x} + c_3 e^{2x}$$

$$y' = -\frac{1}{2}c_2 e^{-\frac{1}{2}x} + 2c_3 e^{2x}$$

$$y'' = \frac{1}{4}c_2 e^{-\frac{1}{2}x} + 4c_3 e^{2x}$$

$$\Rightarrow \begin{cases} 1 = c_1 + c_2 + c_3 \\ -1 = -\frac{1}{2}c_2 + 2c_3 \\ 3 = \frac{1}{4}c_2 + 4c_3 \end{cases}$$

$$\Rightarrow \begin{cases} c_1 = -\frac{7}{2} \\ c_2 = 4 \\ c_3 = \frac{1}{2} \end{cases}$$

$$y = -\frac{7}{2} + 4e^{-\frac{1}{2}x} + \frac{1}{2}e^{2x}$$

3.3.40 $y(x) = Ae^{2x} + B\cos 2x + C\sin 2x$

Characteristic polynomial: $p(r) = (r-2)(r^2+4)$
has roots $2, \pm 2i$ $= r^3 - 2r^2 + 4r - 8$

$p(r) = 0$ is characteristic equation for

$$y^{(3)} - 2y'' + 4y' - 8y = 0$$

~~7/11/17~~