Solutions to 2.6 problems

3) $[0, 0.5]$ $\quad h = 0.25$

$y' = y + 1$

$y(0) = 1$

$y(x) = 2e^x - 1$

$x_0 = 0$

$y_0 = 1$

$k_1 = 1 + 1 = 2$

$k_2 = f\left(\frac{1}{2} \times 0.25, 1 + 0.5 \times 0.25 \times 2\right)$

$\simeq 2.25$

$k_3 = f\left(\frac{1}{2} \times 0.25, 1 + 0.5 \times 0.25 \times 2.25\right)$

$\simeq 2.28$

$k_4 = f(0.25, 1 + 0.25 \times 2.28)$

$k_4 \simeq 2.57$

$y_1 = y_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1 + \frac{0.25}{6}(2 + 2 \times 2.25 + 2 \times 2.28 + 2.57)$

$y_1 \simeq 1.57$ corresponding to $x_1 = 0.25$

Solutions to 2.6 problems:

$x_1 = 0.25$

$y_1 \simeq 1.57$

$k_1 = f(0.25, 1.57)$

$\simeq 2.57$

$k_2 \simeq f(0.25 + 0.5 \times 0.25, 1.57 + 0.5 \times 0.25 \times 2.57) \simeq 2.89$

$k_3 = f(0.25 + 0.5 \times 0.25, 1.57 + 0.5 \times 0.25 \times 2.89)$

$\simeq 2.93$

$k_4 = f(0.5, 1.57 + 0.25 \times 2.93) \simeq 3.30.$

$y_2 \simeq 1.57 + \frac{0.25}{6}(2.57 + 2 \times 2.89 + 2 \times 2.93 + 3.3)$

$y_2 \simeq 2.30$, corresponding to $x_2 = 0.5$

x_n	y_n (Runge-Kutta	y_n (actual)
0	1	1
0.25	1.57	1.57
0.5	2.30	2.30

(They asked for 5 decimal places but, by mistake, I only showed 2 decimal places...)

2.6 problems (continued)

8) $y' = e^{-y}$ $\quad [0, 0.5] \quad h = 0.25$

$y(0) = 0$

$y(x) = \ln(x+1)$

$x_0 = 0$

$y_0 = 0$

$k_1 = f(0,0) = e^{-0} = 1$

$k_2 = f(0.5 \times 0.25, 0.5 \times 0.25 \times 1)$

$= e^{-0.125}$

$\simeq 0.88250$

$k_3 = f(0.5 \times 0.25, 0.5 \times 0.25 \times 0.88250)$

$\simeq 0.89555$

$k_4 = f(0.25, 0.5 \times 0.89555)$

$\simeq 0.63905$

$y_1 = \frac{0.25}{6}(1 + 2 \times 0.88250 + 2 \times 0.89555 + 0.63905)$

$y_1 \simeq 0.21646$ corresponding to $x_1 = 0.25$

2.6 problems (continued)

$x_1 = 0.25$, $y_1 \simeq 0.21646$

$k_1 = f(x_1, y_1) \simeq e^{-0.21646} \simeq 0.80536$

$k_2 = f(0.25 + 0.5 \times 0.25,\ 0.21646 + 0.5 \times 0.25 \times 0.80536)$

$\simeq 0.72824$

$k_3 = f(\sim\sim,\ 0.21646 + 0.5 \times 0.25 \times 0.72824)$

$\simeq 0.73529$

$k_4 = f(\sim,\ 0.21646 + 0.25 \times 0.73529)$

$\simeq 0.67013$

$$y_2 \simeq 0.21646 + \frac{0.25}{6}(0.80536 + 2 \times 0.72824 + 2 \times 0.73529 + 0.67013)$$

$y_2 \simeq 0.39990$ corresponding to $x_2 = 0.5$

x_n	y_n (Runge-Kutta)	y_n (actual)
0	0	0
0.25	0.21646	0.22314
0.5	0.39990	0.40547