

Solutions to 1.2 problems

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$$6) \begin{cases} \frac{dy}{dx} = x\sqrt{x^2+9} \\ y(-4) = 0 \end{cases}$$

$$y = \frac{1}{2} \int 2x\sqrt{x^2+9} dx$$

$$= \frac{1}{2} \cdot \frac{2}{3} (x^2+9)^{\frac{3}{2}} + C$$

$$y = \frac{1}{3} (x^2+9)^{\frac{3}{2}} + C$$

$$y(-4) = 0$$

$$\Rightarrow \frac{1}{3} (16+9)^{\frac{3}{2}} + C = 0$$

$$\Rightarrow C = -\frac{5^3}{3} = -\frac{125}{3}$$

$$\Rightarrow y = \frac{1}{3} (x^2+9)^{\frac{3}{2}} - \frac{125}{3}$$

$$8) \begin{cases} \frac{dy}{dx} = \cos(2x) \\ y(0) = 1 \end{cases}$$

$$y = \frac{1}{2} \sin(2x) + C$$

$$y(0) = 1$$

$$\Rightarrow C = 1$$

$$\Rightarrow y = \frac{1}{2} \sin(2x) + 1$$

$$16) \begin{cases} a(t) = x''(t) = \frac{1}{\sqrt{t+4}} \\ v(0) = -1 \\ x(0) = 1 \end{cases}$$

$$x'(t) = 2\sqrt{t+4} + C$$

$$x'(0) = v(0) = -1$$

$$\Rightarrow 4 + C = -1$$

$$\Rightarrow C = -5$$

$$\Rightarrow x'(t) = 2\sqrt{t+4} - 5$$

$$\Rightarrow x(t) = \frac{2 \cdot 2}{3} (t+4)^{\frac{3}{2}} - 5t + D$$

$$\Rightarrow x(t) = \frac{4}{3} (t+4)^{\frac{3}{2}} - 5t + D$$

$$x(0) = 1$$

$$\Rightarrow \frac{32}{3} + D = 1$$

$$\Rightarrow D = -\frac{29}{3}$$

$$\Rightarrow x(t) = \frac{4}{3} (t+4)^{\frac{3}{2}} - 5t - \frac{29}{3}$$

Solutions to 1.2 problems (continued)

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$$22) \quad x(t) = \begin{cases} \frac{5}{3}t, & \text{if } 0 \leq t \leq 3 \\ 5, & \text{if } 3 \leq t \leq 7 \\ -\frac{5}{3}(t-10), & \text{if } 7 \leq t \leq 10 \end{cases}$$

• $0 \leq t \leq 3$: $\begin{cases} x'(t) = \frac{5}{3}t \\ x(0) = 0 \quad (\text{from graph on p. 18}) \end{cases}$

$$\Rightarrow x(t) = \frac{5}{6}t^2 + C$$

$$x(0) = 0 \Rightarrow C = 0$$

$$\Rightarrow \boxed{x(t) = \frac{5}{6}t^2, \text{ if } 0 \leq t \leq 3}$$

$$\Rightarrow x(3) = \frac{5}{6} \cdot 3^2$$

$$\Rightarrow \boxed{x(3) = \frac{15}{2}}$$

• $3 \leq t \leq 7$: $\begin{cases} x'(t) = 5 \\ x(3) = \frac{15}{2} \end{cases}$

~~$$\Rightarrow x(t) = 5(t-3) +$$~~

$$\Rightarrow x(t) = 5t + D$$

($D \rightarrow$ constant)

$$= 5(t-3) + E$$

(this form is more convenient since we're given $x(3)$)

$$x(3) = \frac{15}{2} \Rightarrow \boxed{E = \frac{15}{2}}$$

$$\Rightarrow \boxed{x(t) = 5(t-3) + \frac{15}{2}, \text{ if } 3 \leq t \leq 7}$$

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by previous formula, $x(7) = 5(7-3) + \frac{15}{2}$

$$\boxed{x(7) = \frac{55}{2}}$$

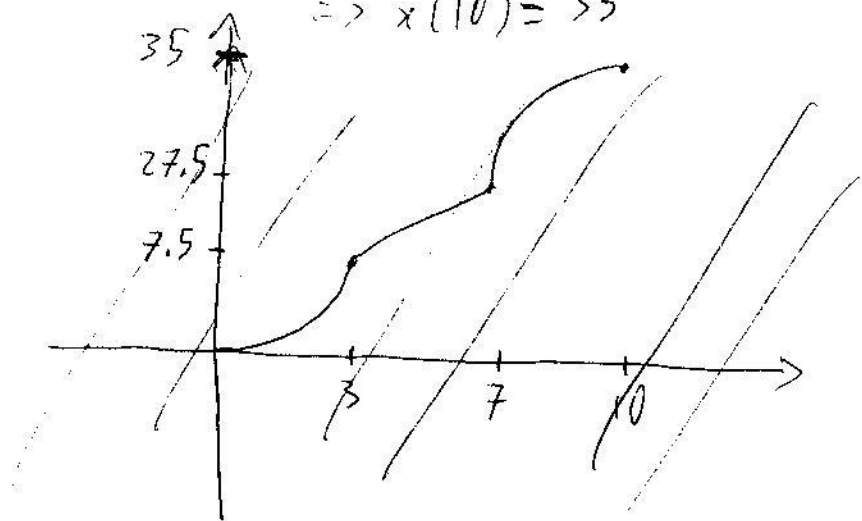
• $7 \leq t \leq 10$: $\begin{cases} x'(t) = -\frac{5}{3}(t-10) \\ x(7) = \frac{55}{2} \end{cases}$

$$x'(t) = -\frac{5}{3}t + \frac{50}{3}$$

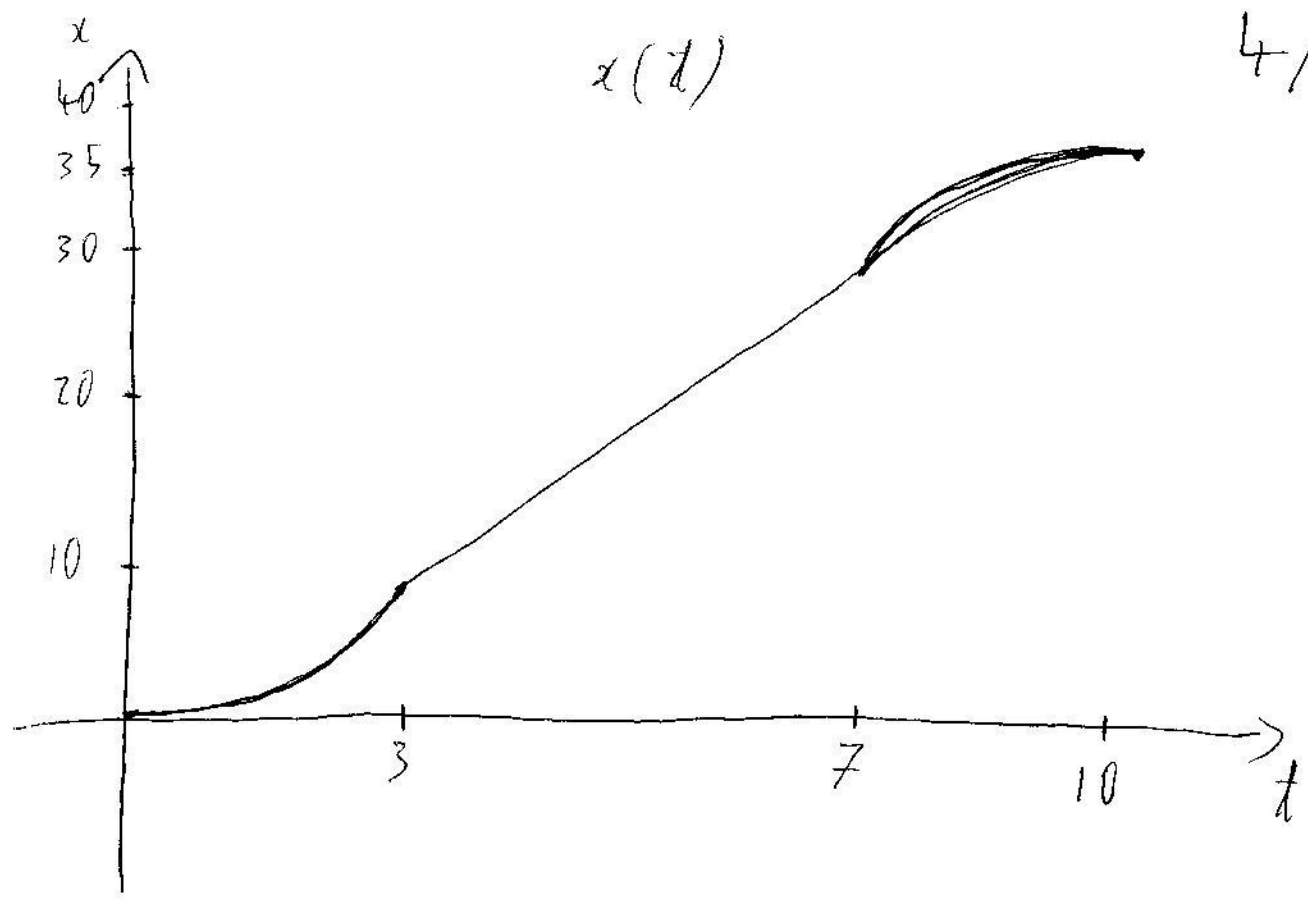
$$\Rightarrow x(t) = -\frac{5}{6}t^2 + \frac{50}{3}t + F = -\frac{5}{6}(t^2 - 20t + H)$$

$$x(7) = \frac{55}{2} \Rightarrow H = 58 \Rightarrow x(t) = -\frac{5}{6}(t^2 - 20t + 58)$$

$$\Rightarrow x(10) = 35$$



$x(t)$



problem 22 (sec. 1.2)