Verify by substitution that y_1 and y_2 are solutions.

1.1.6. $y'' + 4y' + 4y = 0; \quad y_1 = e^{-2x}, \quad y_2 = xe^{-2x}$

Solution

$y_1' = -2e^{-2x}$ $\qquad$ $y_2' = e^{-2x}(1-2x)$

$y_1'' = +4e^{-2x}$ $\qquad$ $y_2'' = e^{-2x}(4x-4)$

$$\begin{aligned} y_1'' + 4y_1' + 4y_1 &= (4e^{-2x}) + 4(-2e^{-2x}) + 4(e^{-2x}) \\ &= 4e^{-2x} - 8e^{-2x} + 4e^{-2x} \\ &= 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} y_2'' + 4y_2' + 4y_2 &= (e^{-2x}(4x-4)) + 4(e^{-2x}(1-2x)) + 4(xe^{-2x}) \\ &= 4xe^{-2x} - 4e^{-2x} + 4e^{-2x} - 8xe^{-2x} + 4xe^{-2x} \\ &= 0 \end{aligned}$$

1.1.10 $x^2y'' + xy' - y = \ln x$

$y_1 = x - \ln x, \quad y_2 = \frac{1}{x} - \ln x$

Solution:

$y_1' = 1 - \frac{1}{x}$ $\qquad$ $y_2' = \frac{-1}{x^2} - \frac{1}{x}$

$y_1'' = \frac{1}{x^2}$ $\qquad$ $y_2'' = \frac{2}{x^3} + \frac{1}{x^2}$

$$\begin{aligned} x^2y_1'' + xy_1' - y_1 &= x^2\left(\tfrac{1}{x^2}\right) + x\left(1 - \tfrac{1}{x}\right) - (x - \ln x) \\ &= \ln x \end{aligned}$$

$$\begin{aligned} x^2y_2'' + xy_2' - y_2 &= x^2\left(\tfrac{2}{x^3} + \tfrac{1}{x^2}\right) + x\left(-\tfrac{1}{x^2} - \tfrac{1}{x}\right) - \left(\tfrac{1}{x} - \ln x\right) \\ &= \ln x \end{aligned}$$

1.1.16 Substitute $y = e^{rx}$ into the differential equation to determine all values of r for which $y = e^{rx}$ is a solution.

$$3y'' + 3y' - 4y = 0.$$

Solution:

$$y' = re^{rx}$$
$$y'' = r^2 e^{rx}$$

$$3y'' + 3y' - 4y = 0 \Rightarrow 3(r^2 e^{rx}) + 3(re^{rx}) - 4e^{rx} = 0$$
$$\Rightarrow (3r^2 + 3r - 4)e^{rx} = 0$$
$$\Rightarrow 3r^2 + 3r - 4 = 0$$
$$\Rightarrow r = \frac{-1}{2} \pm \frac{\sqrt{57}}{6}$$

1.1.24

Verify that $y(x)$ satisfies the DE. Then determine the constant C so that $y(x)$ satisfies the initial condition.

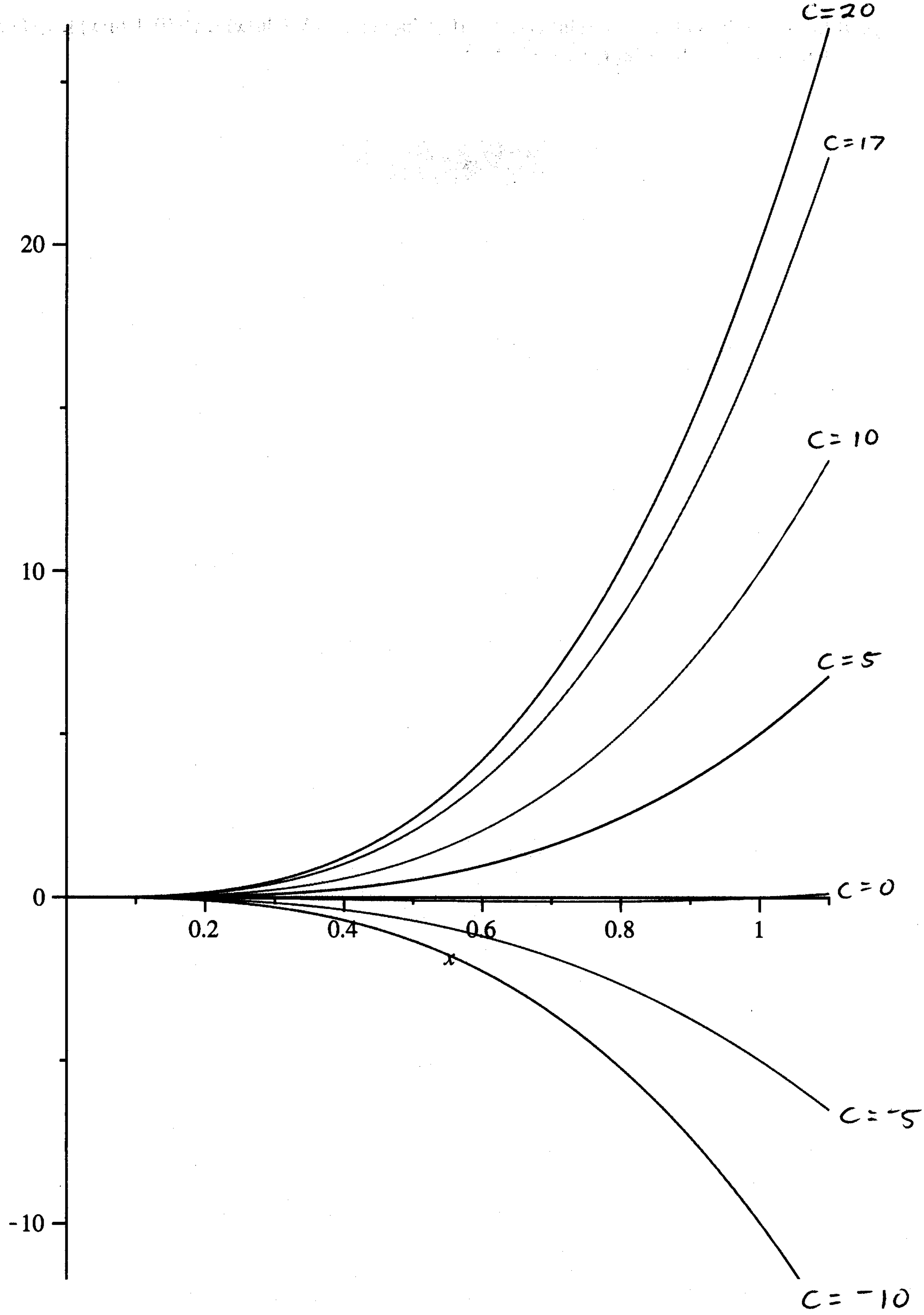
$$xy' - 3y = x^3 \quad ; \quad y(x) = x^3(C + \ln x), \quad y(1) = 17$$

Solution: $y' = 3x^2(C + \ln x) + x^2$

$$xy' - 3y = x(3x^2(C + \ln x) + x^2) - 3(x^3(C + \ln x))$$
$$= x^3$$

$$y(1) = 17 \Rightarrow 1^3(C + \ln 1) = 17$$
$$\Rightarrow C = 17$$

1.1.24



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1.1.34

The acceleration $\frac{dv}{dt}$ of a car is proportional to the difference between 250 km/h and the velocity of the car.

$$\frac{dv}{dt} = k(250 - v), \quad \text{for some constant } k.$$

1.1.40

Determine by inspection at least one solution to $(y')^2 + y^2 = 1$.

$y_1(x) = 1$ and $y_2(x) = -1$ are solutions since

$y_1'(x) = y_2'(x) = 0$ and $[y_1(x)]^2 = [y_2(x)]^2 = 1$.