MAT303 Spring 2009

Practice Midterm II

The actual final will consist of five problems with no more then two subproblems

You will be allowed to use calculators

Problem 1.

- Find and sketch the equilibrium solutions of the ODE

$$y^{'} = (y+3)^2(y+1)(y-3).$$

- On the same plot, sketch at least one solution curve of this ODE in each region of the x, y-plane cut out by the graphs of the equilibrium solutions. Indicate the asymptotic behavior. Determine whether each of the equilibrium solutions is asymptotically stable or unstable. Draw the phase line.

Problem 2.

Consider the autonomous equation $y^{'} = \sin y$.

- What are the equilibrium solutions for this equation?
- Which of these equilibrium solutions are stable, which are unstable? Sketch a set of integral curves to indicate the general behavior of this system.
- Suppose $y = y_1(x)$ is the solution with initial value $y(0) = 10$. What is $\lim_{x\to+\infty} y(x)$?

Problem 3.

- Find critical points of the differential equation

$$y^{'} = (y-1)(y+3) + c$$

- Find bifurcation points c.

- Find stable and unstable critical points

- Draw the bifurcation diagram

Problem 4.

- If the solution curves to a differential equation are all concave up, would Eulers method starting at a point (x_0, y_0) give an under approximation or an over approximation to the solution curve passing through (x_0, y_0)? Justify your answer with a picture of a slope field.

- Use the improved Euler method with step-size 1 to estimate $y(2)$ for the initial value problem:

$$y^{'} = xy \quad y(1) = 2$$

Problem 5. For what value of the constant c the differential equation $y^{''} + cy^{'} + 10y = 0$ is underdamped, critically damped, or overdamped.

- Sketch a typical solution in the critically damped case.
- Pick c such the system is underdamped. Find a general solution.
- Rewrite the function $\cos(2x) + \sqrt{3}\sin(2x)$ in the form $A\cos(\omega x - b)$.
- Write the general solution un the underdapmed case in the form $Ae^{rt}\cos(\omega t - b)$ for suitable A, b, ω, b.

Problem 6. Solve the following Initial Value Problem using the method of indeterminate coefficients.

1. $y^{''} - 2y^{'} + y + xe^{x} = 0 \quad y(0) = 1, y^{'}(0) = 1$
2. $y^{''} + 4y + \cos(2x) = 0 \quad y(0) = 0, y^{'}(0) = 0$

Problem 7.

- Show that the functions $\cos(x) + 3\sin(x)$ and $\sin(x)$ are linearly independent functions on the real line by looking at their Wronskian.
- Find the general solution of the following equation using the method of variation of parameter
$$y^{''} + y + 1 + x = 0$$

Problem 8.

A mass of 2kg is attached to an undamped spring (no gravity). The spring has spring constant of 3kg/sec^2 and is at rest. At time $t = 1$, the spring is hit with a hammer, imparting a unit impulse force. Find the equation of the resulting motion and solve it.