Solution to practice final:

5) $\frac{dx}{dt} = x(2-x) - h$ h small

⚠ By mistake I solved the problem without the $\frac{1}{2}$. Sorry...

Equilibrium solutions:

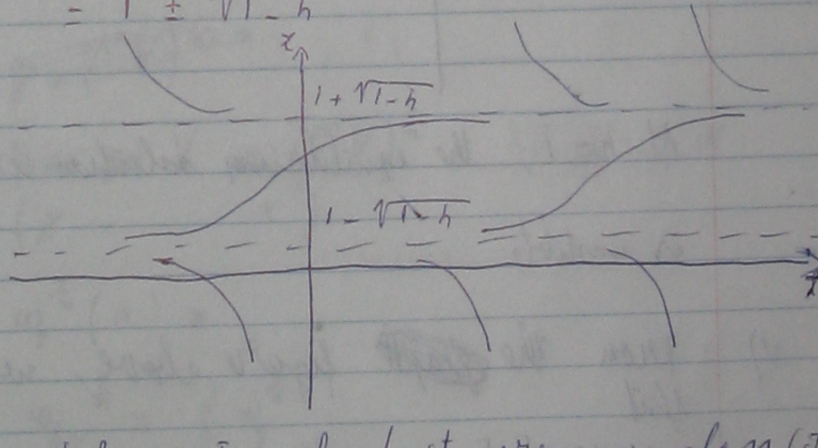
We solve for $x_{eq}(2 - x_{eq}) - h = 0$

$$x_{eq}^2 - 2x_{eq} + h = 0$$

$\Delta = 4 - 4h > 0$ (since h is small, so we assume that $h < 1$)

$$\Rightarrow x_{eq} = \frac{2 \pm \sqrt{4-4h}}{2}$$

$$= 1 \pm \sqrt{1-h}$$



⚠ I believe I solved it wrong in class (Joe).

Thus $x = 1 + \sqrt{1-h}$ is stable

$x = 1 - \sqrt{1-h}$ is unstable.

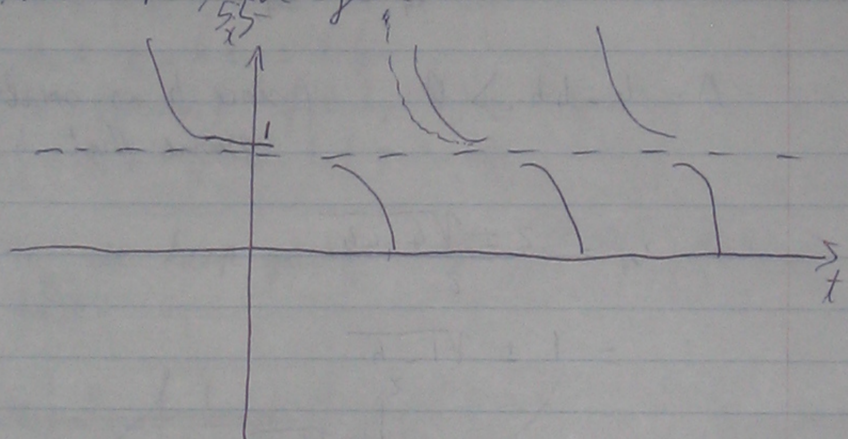
Solution to practice final

5 (continued)

iii) bifurcation point, when $\Delta = 0$,

i.e. $4 - 4h = 0 \iff h = 1$

At $h = 1$, we get:



At $h=1$, the equilibrium solution $x = 1$

is unstable

v) From the ~~graph~~ figure above, we see that

a) $u(2) = 5.5 \Rightarrow \lim_{t\to\infty} u(t) = 1$

b) $u(0) = 2 \Rightarrow \lim_{t\to\infty} u(t) = 1$

c) $u(0) = 1 \to$ Apply Euler's method.