Pract. final solution:

Prob. 3) $\underbrace{(3x-y)}_{P(x,y)}dx \underbrace{-(x+3y)}_{Q(x,y)}dy = 0$

$\frac{\partial P}{\partial y} = -1$

$\frac{\partial Q}{\partial x} = -1$

so ODE is exact.

Hence there is an $f(x,y)$ st

$$\begin{cases} \frac{\partial f}{\partial x} = P = 3x - y & (1) \\ \frac{\partial f}{\partial y} = Q = -(x+3y) & (2) \end{cases}$$

From (1), ~~by~~ integrating with respect to x:

$f = \frac{3}{2}x^2 - xy + g(y)$

We differentiate with respect to y and compare with (2):

$\frac{\partial f}{\partial y} = -x + g'(y) = -(x+3y)$

$\Rightarrow g'(y) = -3y \Rightarrow g(y) = -\frac{3}{2}y^2 + C$

$\Rightarrow$ gen. solution given by f = constant, i.e.

$$\boxed{\frac{3}{2}x^2 - xy - \frac{3}{2}y^2 = D}$$

Prob. 3 (continued):

$$\underbrace{(3x^2 - xy)}_{A(x,y)}dx - \underbrace{(x^2+3xy)}_{B(x,y)}dy = 0 \qquad x \neq 0$$

$$\frac{\partial A}{\partial y} = -x \quad , \quad \frac{\partial B}{\partial x} = -2x+3y$$

This ODE is not exact BUT, if we divide by x (since $x \neq 0$, we can do that), it becomes exact:

$$\underbrace{(3x-y)}_{P(x,y)}dx - \underbrace{(x+3y)}_{Q(x,y)}dy = 0$$

$$\left.\begin{matrix}\frac{\partial P}{\partial y} = -1 \\ \\ \frac{\partial Q}{\partial x} = -1\end{matrix}\right\} \rightarrow \text{exact}$$

$$\frac{\partial f}{\partial x} = 3x - y \Rightarrow f = \frac{3}{2}x^2 - xy + g(y)$$

We differentiate w.r.t. y and set it equal to Q:

$$\frac{\partial f}{\partial y} = -x + g'(y) = -x-3y \Rightarrow g'(y) = -3y$$

$$\Rightarrow g(y) = -\frac{3}{2}y^2 + C$$

gen. solution: $\boxed{\frac{3}{2}x^2 - xy - \frac{3}{2}y^2 = D}$