MAT-126 H.W. #6

pgs. 392-393 #2, 4, 6, 8, 10, 14, 16, 22, 24, 30, 34, 40, 44, 48, 52, 64

Substitution Rule for <u>indefinite</u> integrals: $\int f(g(x))g'(x)dx = \int f(u)du$; $u = g(x)$, $du = g'(x)dx$

2-) $\int x(4+x^2)^{10}dx$, $u = 4+x^2$

Since $u = 4+x^2$, then $du = 2x\,dx \to x\,dx = \frac{1}{2}du$

$\int x(4+x^2)^{10}dx = \frac{1}{2}\int u^{10}du = \frac{u^{11}}{22} + C = \frac{(4+x^2)^{11}}{22} + C$

4-) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$, $u = \sqrt{x}$

Since $u = \sqrt{x} = x^{1/2}$, then $du = \frac{1}{2\sqrt{x}}dx \to \frac{1}{\sqrt{x}}dx = 2du$

$2\int \sin u\,du = -2\cos u + C = -2\cos\sqrt{x} + C$

6-) $\int e^{\sin\theta}\cos\theta\,d\theta$, $u = \sin\theta$

$u = \sin\theta \to du = \cos\theta\,d\theta$

Hence, $\int e^u du = e^u + C = e^{\sin\theta} + C$

8-) $\int x^2(x^3+5)^9 dx$

$u = x^3+5 \to du = 3x^2dx \to x^2dx = \frac{1}{3}du$

$\frac{1}{3}\int u^9 du = \frac{u^{10}}{30} + C = \frac{(x^3+5)^{10}}{30} + C$

10-) $\int xe^{x^2}dx$

$u = x^2 \to du = 2x\,dx \to x\,dx = \frac{1}{2}du$

$\frac{1}{2}\int e^u du = \frac{e^u}{2} + C = \frac{e^{x^2}}{2} + C$

14-) $\int \frac{x}{(x^2+1)^2}dx$

$u = x^2+1 \to du = 2x\,dx \to x\,dx = \frac{1}{2}du$

$\frac{1}{2}\int \frac{1}{u^2}du = -\frac{1}{2u} + C = -\frac{1}{2(x^2+1)} + C$

16-) $\int \frac{1}{(5t+4)^{2.7}}dt$

$u = 5t+4 \to du = 5dt \to dt = \frac{1}{5}du$

$\frac{1}{5}\int \frac{1}{u^{2.7}}du = -\frac{1}{5}\cdot\frac{1}{1.7u^{1.7}} + C = -\frac{1}{8.5(5t+4)^{1.7}} + C$

22-) $\int \frac{x}{x^2+1}dx$

$u = x^2+1 \to du = 2x\,dx \to x\,dx = \frac{1}{2}du$

$\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\underbrace{\ln u}_{} + C = \frac{1}{2}\ln(x^2+1) + C$

Using $\ln|u|$ isn't necessary, since $u>0$ $(x^2+1>0)$.

24-) $\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$

$u=\tan^{-1}(x) \rightarrow du=\frac{1}{1+x^2}\,dx$

$\int u\,du = \frac{u^2}{2}+C = \frac{(\tan^{-1}(x))^2}{2}+C$

30-) $\int \frac{e^x}{e^x+1}\,dx$

$u=e^x+1 \rightarrow du=e^x\,dx$

$\int \frac{1}{u}\,du = \ln u + C = \ln(e^x+1)+C$; $\ln|e^x+1|$ wasn't necessary, since $e^x+1>0$

34-) $\int \frac{x}{1+x^4}\,dx = \int \frac{x}{(1+x^2)^2-2x^2}\,dx$

$u=x^2 \rightarrow du=2x\,dx \rightarrow x\,dx=\frac{1}{2}du$

$\frac{1}{2}\int \frac{du}{(1+u)^2-2u} = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{\tan^{-1}(u)}{2}+C = \frac{\tan^{-1}(x^2)}{2}+C$

Substitution Rule for <u>definite</u> integrals: $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$; $u=g(x)$; $du=g'(x)\,dx$

40-) $\int_0^7 \sqrt{4+3x}\,dx$

$u=4+3x \rightarrow du=3dx \rightarrow dx=\frac{1}{3}du$

at $x=7 \rightarrow u=25$

at $x=0 \rightarrow u=4$

$\frac{1}{3}\int_4^{25} u^{1/2}du = \frac{2}{9}u^{3/2}\Big|_4^{25} = \frac{2}{9}\left(\sqrt{(25)^3}-\sqrt{(4)^3}\right)=26$

44-) $\int_{1/6}^{1/2} \csc\pi t\,\cot\pi t\,dt$

$u=\pi t \rightarrow du=\pi\,dt \rightarrow dt=\frac{1}{\pi}du$

at $t=\frac{1}{2} \rightarrow u=\frac{\pi}{2}$

at $t=\frac{1}{6} \rightarrow u=\frac{\pi}{6}$

$\frac{1}{\pi}\int_{\pi/6}^{\pi/2} \csc u\cot u\,du = \frac{1}{\pi}(-\csc u)\Big|_{\pi/6}^{\pi/2} = \frac{1}{\pi}\left(-\frac{1}{\sin u}\right)\Big|_{\pi/6}^{\pi/2} = \frac{1}{\pi}(-1+2)=\frac{1}{\pi}$

48-) $\int_{-\pi/2}^{\pi/2} \frac{x^2\sin x\,dx}{1+x^6}$

"$f(x)=\frac{x^2\sin x}{1+x^6}$" is continuous on $[-\frac{\pi}{2},\frac{\pi}{2}]$ & $f(x)$ is an odd function, since $\left.\begin{array}{l} f(-x)=\frac{x^2\overbrace{\sin(-x)}^{-\sin x}}{1+x^6} \\ -f(x)=\frac{-x^2\sin x}{1+x^6}\end{array}\right\}$ $f(-x)=-f(x)$

Hence $\int_{-\pi/2}^{\pi/2} \frac{x^2\sin x\,dx}{1+x^6}=0$

52-) $\int_0^a x\sqrt{a^2-x^2}\,dx$

$u=a^2-x^2 \to du=-2x\,dx \to x\,dx=-\frac{1}{2}du$; $x=0 \to u=a^2$; $x=a \to u=0$

$-\frac{1}{2}\int_{a^2}^{0} u^{1/2}du = \frac{1}{2}\int_0^{a^2} u^{1/2}du = \frac{2}{3}\left(\frac{1}{2}\right)u^{3/2}\Big|_0^{a^2} = \frac{a^3}{3}$

64-) Using $u=x^2 \to du=2x\,dx \to x\,dx=\frac{1}{2}du$; $u=0$, when $x=0$; $u=9$, when $x=3$

$\int_0^3 x f(x^2)\,dx = \frac{1}{2}\underbrace{\int_0^9 f(u)\,du}$

Using $u=x$, $du=dx$ $\to \frac{1}{2}\int_0^9 f(x)\,dx = \frac{1}{2}(4)=2$