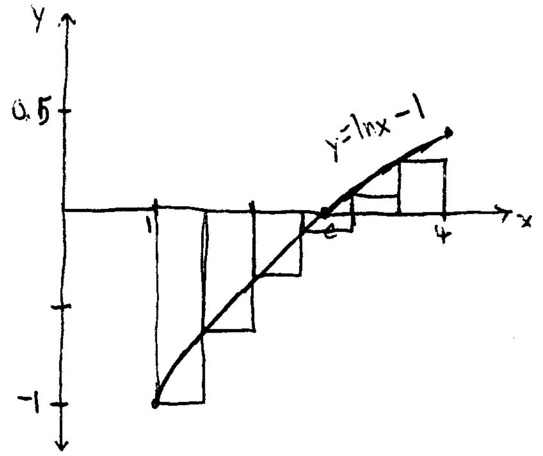
MAT-126 H.W. #2 Solutions:

5.2 # 2, 12, 18, 20, 28, 32, 34, 42, 44

2-)

Let's say that $g(x) = \ln x$, then $f(x) = \ln x - 1$ will take the graph of $g(x)$ and shift it 1 unit downward.



- Looking at the graph of $f(x)$, we notice that we can't find the area of $f(x)$, but we can certainly find the "difference in areas" for $[1,4]$, using left endpoints.

$$L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x ; \Delta x = \frac{b-a}{n} = 0.5$$

$$L_6 = 0.5\left(f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5)\right)$$

$$L_6 = 0.5(-1.63372172) \approx -0.816861$$

Riemann Sum represents: $\begin{pmatrix}\text{Sum of the areas of} \\ \text{the two rectangles that} \\ \text{are "above" the x-axis}\end{pmatrix} - \begin{pmatrix}\text{Sum of the areas of} \\ \text{the four rectangles that} \\ \text{are "below" the x-axis}\end{pmatrix}$

12-) Recalling the "Midpoint Rule" on pg. 360, we have: (using $n=4$)

$$\int_1^5 x^2 e^{-x}\,dx \approx \Delta x\left[f(\bar{x}_1)+\cdots+f(\bar{x}_n)\right]; \quad \Delta x=\frac{b-a}{n}=1 \text{ and } \bar{x}_i=\tfrac{1}{2}(x_{i-1}+x_i)$$

$$\int_1^5 x^2 e^{-x}\,dx \approx f(1.5)+f(2.5)+f(3.5)+f(4.5); \quad \text{midpoints are: } \bar{x}_1=1.5,\ \bar{x}_2=2.5,\ \bar{x}_3=3.5,\ \bar{x}_4=4.5$$

$$\int_1^5 x^2 e^{-x}\,dx \approx (1.5)^2e^{-1.5}+(2.5)^2e^{-2.5}+(3.5)^2e^{-3.5}+(4.5)^2e^{-4.5}\approx 1.6099$$

Conclusion: Since $f(x)\geq 0$ for $1\leq x\leq 5$, where $f(x)=x^2e^{-x}$; the value 1.6099 is the "area" under the curve of $f(x)=x^2e^{-x}$ for $1\leq x\leq 5$

18-) Using definition 3: $\left(\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x\right)$, we have:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{e^{x_i}}{1+x_i}\Delta x=\int_1^5\frac{e^x}{1+x}\,dx; \quad f(x)=\frac{e^x}{1+x},\ a=1 \text{ and } b=5.$$

Conclusion: The definite integral of "$\frac{e^x}{1+x}$" from 1 (lower limit) to 5 (upper limit) is: $\boxed{\int_1^5\frac{e^x}{1+x}\,dx=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{e^{x_i}}{1+x_i}\Delta x}$

20-) If we replace "$\lim\Sigma$" by "$\int$", "x_i^*" by x, and Δx by dx, we are left with:

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x=\int_a^b f(x)\,dx$$

Using this methodology: $(f(x)=4-3x^2+6x^5)$

The definite integral of "$4-3x^2+6x^5$" from 0 (lower limit) to 2 (upper limit) is: $\boxed{\int_0^2(4-3x^2+6x^5)\,dx=\lim_{n\to\infty}\sum_{i=1}^{\infty}\left(4-3(x_i^*)^2+6(x_i^*)^5\right)\Delta x}$

28-) Using the same mentality as problems 18, 20:

$$\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x; \quad \Delta x=\frac{b-a}{n} \text{ and } x_i=a+i\Delta x$$

$$\int_1^{10}(x-4\ln x)\,dx=\lim_{n\to\infty}\sum_{i=1}^{n}(x_i-4\ln x_i)\Delta x; \quad \Delta x=\frac{10-1}{n}=\frac{9}{n} \text{ and } x_i=1+i\Delta x$$

$$\boxed{\underbrace{\int_1^{10}(x-4\ln x)\,dx}_{\text{definite integral}}=\lim_{n\to\infty}\underbrace{\sum_{i=1}^{n}\left(\left(1+\frac{9i}{n}\right)-4\ln\left(1+\frac{9i}{n}\right)\right)\left(\frac{9}{n}\right)}_{\text{Riemann Sum}}}$$

32-) Using the Knowledge that we absorbed Exs. 4a, 4b:

a-) Find the area of this right Δ with base $b=2$ and height $h=4$:

$\rightarrow$ Area $= \frac{1}{2}bh = \frac{1}{2}\cdot 8 = 4 \rightarrow$ Hence $\boxed{\int_0^2 g(x)dx = 4}$

b-) Find the area of this semicircle with radius $r=2$ (radius is found by drawing a vertical line from $(4,0)$ to $(4,2)$)

$\rightarrow$ Area $= 2\cdot\frac{1}{4}\pi r^2 = \frac{1}{2}\pi r^2 = 2\pi \rightarrow$ Hence $\boxed{\int_2^6 g(x)dx = 2\pi}$

c-)

$$\left[(\text{Area from } x=0 \text{ to } x=2) + (\text{Area from } x=6 \text{ to } x=7)\right] - \left[\text{Area from } x=6 \text{ to } x=7\right] = \int_0^7 g(x)\,dx$$

Area from $x=6$ to $x=7$:

Area of right Δ is: "$\frac{1}{2}bh = \frac{1}{2}(1)(1) = \frac{1}{2}$"

Hence, $\int_6^7 g(x)dx = \frac{1}{2}$

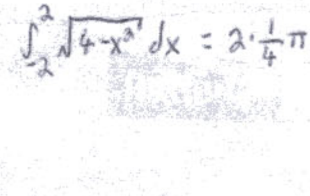
Using 32a, 32b, we now have:

$$\int_0^7 g(x)dx = 4 + \frac{1}{2} - 2\pi = \frac{9}{2} - 2\pi$$

34-) Using $x^2+y^2=r^2$ (center at origin):

$x^2+y^2=4 \rightarrow y=\sqrt{4-x^2}$; radius $r=2$ (Note: "$-$" sign is omitted, since only $f(x)=\sqrt{4-x^2}$ is given)

Since y is non-negative, the graph looks like ($f(x)\geq 0$)



The area under the curve "$y=\sqrt{4-x^2}$" from -2 (lower limit) to 2 (upper limit) is:

$$\int_{-2}^{2}\sqrt{4-x^2}\,dx = 2\cdot\frac{1}{4}\pi r^2 = \frac{1}{2}\pi(2)^2 = 2\pi$$

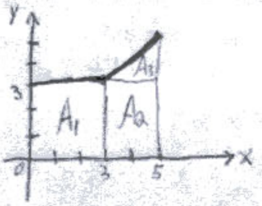
42-) Equation 5 says: $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$;

$$\int_1^5 f(x)\,dx = \int_1^4 f(x)\,dx + \int_4^5 f(x)\,dx$$

$$\int_1^4 f(x)\,dx = \int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx$$

Hence, the area of $f(x)$ from $x=1$ to $x=4$ is ; $12-3.6=8.4$

44-) Graphing this "piecewise" function, we get ;



$$\int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx \text{ ;}$$

- $\int_0^3 f(x)\,dx = \text{Area of rectangle } (A_1) = \int_0^3 3\,dx$

Using formula 1 on pg. 361 : $\int_0^3 3\,dx = 3(3-0) = 9$

- $\int_3^5 f(x)\,dx = \text{Area of rectangle } (A_2) + \text{Area of } \overset{\text{right}}{\Delta}\ (A_3)$

$$= 2\cdot 3 + \frac{1}{2}(2)(2) = 8$$

Therefore, $\boxed{\int_0^5 f(x)\,dx = 9+8 = 17}$