

**MAT313 Fall 2008**

**Practice Midterm II**

**Problem 1.** Let  $X$  be a set of lines through 0 in  $\mathbb{R}^2$ . The group  $SL_2(\mathbb{R})$  acts on  $X$  by linear transformations. Let  $H$  be the stabilizer of a line defined by the equation  $y = 0$ .

1. Describe the set of matrices  $H$ .
2. Describe the orbits of  $H$  in  $X$ . How many orbits are there?
3. Identify  $X$  with the set of cosets of  $SL_2(\mathbb{R})$ .

**Problem 2.**

1. Describe all elements of order eight in  $\mathbb{Q}/\mathbb{Z}$ .
2. Find all elements of infinite order in  $\mathbb{Q}/\mathbb{Z}$ .
3. Identify  $\mathbb{Q}/\mathbb{Z}$  with a subgroup of  $\mathbb{C}^*$

**Problem 3.**

Give an example of a non commutative group with a normal subgroup of index  $p - 1$ , where  $p$  is prime

**Problem 4.** Give an example of a non commutative group that contains a subgroup of prime order.

**Problem 5.** Let  $G$  be the group of quaternions, i.e.,  $G = \{1, -1, i, j, k, (-1)i, (-1)j, (-1)k\}$ . The elements  $1, -1$  are central and satisfy  $-1^2 = 1$ . In addition  $i^2 = j^2 = k^2 = -1$ ,  $(-1)ji = ij = k, (-1)ki = ik = (-1)j, (-1)jk = kj = (-1)i$ .

Find orders of all elements in  $G/Z(G)$ , Is  $G/Z(G)$  isomorphic to  $\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_2$ . Why?

**Problem 6** Give a definition of a factor group.