**MAT313 Fall 2008**

## Practice Midterm II

You are allowed to use calculators and books.

**Problem 1.**

1. Find the order of the group $U(20)$.

2. Find the order of each element in the group $U(20)$ and determine whether $U(20)$ is cyclic or not.

**Problem 2.**

1. Let $G =< a >$ be a cyclic group of order $n$, and let $k$ be a positive integer. Find the order of the element $a^k$.

2. Let $G = \mathbb{Z}_{90000}$. Using part 1, find all elements of order 9 in $G$.

**Problem 3.** Let $G$ be a group and let $x, y \in G$ be elements such that $|y| = 2$, $x \neq 1$ (1 is the unit) and

$$yxy = x^2$$

1. Using the above equation show that $(yx)^2 = x^3$.

2. Using the above equation show that $yx = xyx^{-1}$ (Hint: multiply the above equation on $x^{-1}$ from the right and use $|y| = 2$).

3. Prove that $|xyx^{-1}| = |y| = 2$ and find $|yx|$ using part 1.

4. Find $|x|$ using parts 1,3.

**Problem 4.**

Consider the permutation $\sigma = (1235)(2467)$

1. Find $\sigma(4)$ and $\sigma^2(4)$.

2. Find $\sigma^{-1}$.

3. Write $\sigma$ as a product of disjoint cycles.

4. Find the order of $\sigma$.

**Extra Credit**

1. Let $G$ be a group, let $H$ be its subgroup and let $a \in G$ be an element of order $n$. Prove that if $a^n \in H$ and $m$ and $n$ are relatively prime then $a \in H$.

2. Let $x, y \in G$ and $xy \in C(x)$ the centralizer of $x$ in $G$. Prove that $xy = yx$.