MAT313 Fall 2008

Practice Final

The actual final will consist of ten problems

Solutions only ten randomly chosen problems will be posted

Problem 1.

Consider a strip of equally spaced letters

$$\cdots - \mathrm{O} - \mathrm{O} - \mathrm{O} - \mathrm{O} - \cdots$$

Describe the symmetry group of the strip. Is the group abelian?

Problem 2.

Give four non isomorphic examples of groups of order eight. You must explain why the groups are mutually non isomorphic.

Problem 3.

Find a group that contains elements a, b such that $|a| = |b| = 2$ and

1. $|ab| = 3$
2. $|ab| = 4$
3. $|ab| = 30$

Problem 4.

Suppose H is a proper subgroup of $\mathbb{Z}$ under addition and H is generated by $18, 30$ and 40. Determine H.

Problem 5.

The group $U(5)$ has five cyclic subgroups. List them.

Problem 6.

List all elements of $\mathbb{Z}_{40}$ that have order ten.

Problem 7.

Suppose $|x| = n$. Find a necessary and sufficient condition on s and t such that $(x^t) \subset (x^s)$.

Problem 8.

Determine whether the following permutations are even or odd.

- (135)
- (1356)
- (13567)
- (12)(134)(152)
- (1243)(3521)

Problem 9. What is the order of

- (124)(357)
- (124)(35)
- (345)(245)

Problem 10.

Compute the centraliser of (12)(34) in S_4.

Problem 11.

Prove that $U(20)$ and $U(24)$ are isomorphic.

Problem 12.

Prove that the group of nonzero complex number under multiplication is not isomorphic to the group of complex numbers under addition.

Problem 13.

Let S be a set of subgroups of order p in $\mathbb{Z}_p \oplus \mathbb{Z}_p$. Show that $\mathrm{GL}(2, \mathbb{Z}_p)$ acts on S. Choose an element $m \in S$. Find the stabiliser of m in $\mathrm{GL}(2, \mathbb{Z}_p)$. Find the orbit of m.

Problem 14.

Prove that the factor group of abelian group is abelian.

Problem 15.

Let H be a normal subgroup of G and a be an element of G. If the element aH has order 3 in G/H and $|H| = 10$ what is the possibilities for the order of a.

Problem 16.

Suppose $\mathbb{Z}_{10}$ and $\mathbb{Z}_{15}$ are homomorphic images of the group G. What can we say about $|G|$.

Problem 17.

Determine all homomorphisms of $\mathbb{Z}$ onto S_3. Determine all homomorphisms of $\mathbb{Z}$ to S_3.

Problem 18.

Find all idempotents in $\mathbb{Z}_5[i] = \{a + bi | a, b \in \mathbb{Z}_5\}$.

Problem 19.

Prove that the ring $\{a + b\sqrt[3]{7} + c\sqrt[3]{49} | a, b, c \in \mathbb{Q}\}$ is a field.

Problem 20.

Find all solutions of $x^2 - x + 2 = 0$ in $\mathbb{Z}_3[i]$.

Problem 21.

Show that $\mathbb{R}[x]/(x^2 - 2)$ is not a field, but $\mathbb{Q}[x]/(x^2 - 2)$ is.

Problem 22.

Find all the ring homomorphisms $\psi : \mathbb{R} \to \mathrm{Mat}_2(\mathbb{R})$ and $\phi : \mathrm{Mat}_2(\mathbb{R} \to \mathbb{R}$. ψ and ϕ are homomorphisms of rings with units, i.e. $\psi(1) = 1$ and $\phi(1) = 1$.

Problem 23.

Do homomorphisms of ring form a group with respect to addition. Explain.