HW8

This is due Friday, April 26

1. Evaluate the infinite continued fractions:

   a $< 2, 2, \cdots >$

   b $< 2, 1, 2, 1, \cdots >$

   c $< 1, 2, 1, 2, \cdots >$

   d $< 1, 3, 1, 3, \cdots >$

2. For $n \geq 1$, prove that $k_n/k_{n-1} =< a_n, \ldots, a_1 >$(See Sec. 7.3 of the book for notations). Find and prove a similar continued fraction expansion for h_n/h_{n-1}, assuming $a_0 \geq 0$.

3. Expand each of the following as infinite continued fractions: $\sqrt{2}$, $\sqrt{2}-1$, $\sqrt{2}/2$, $1/\sqrt{3}$.

4. Given that two irrational numbers have identical convergents $h_0/k_0, h_1/k_1, \ldots$ up to h_n/k_n, prove that their continued fraction expansions are identical up to a_n.

5. Let ξ be an irrational number. In the notations of Sec. 7.3 prove that for $n \geq 1$,

   $$\xi - \frac{h_n}{k_n} = (-1)^n k_n^{-2} (\xi_{n+1} + < 0, a_n, a_{n-1}, \ldots, a_1 >)^{-1}$$

   (Hint :use results of the second problem in combination with Theorem 7.5, Sec. 7.3)

6. Let $a_0, a_1, \ldots, a_n$ and $b_0, b_1, \ldots, b_{n+1}$ be positive integers. State the condition for

   $$< a_0, a_1, \ldots, a_n > < < b_0, b_1, \ldots, b_{n+1} > .$$