HW3

This is due Friday Feb. 22

1. Show that if p is a prime then $(p-1)! \equiv p-1 \mod 1+\ldots+p-1$.

2. Show that $7|(3^{2n+1}+2^{n+2})$ for all $n \geq 0$.

3. Prove that $n^{12}-a^{12}$ is divisible by 91 if a, n are relatively prime to 91.

   (Hint: factor 91 first. Solve the analogous problem modulo the factors. Then deduce the proof.)

4. Determine weather the congruences $5x \equiv 1 \mod 6$, $4x \equiv 13 \mod 15$ have common solutions and find them if they exist.

5. Let $\phi(n)$ be the Euler's function. Compute $\phi(p^k)$ where p is a prime.

6. Construct a homomorphism from the group of integers (with the operation of addition) to the group of nonzero rational numbers(with the operation of multiplication).