Practice Midterm II

The actual midterm will consist of four problems.

Calculators will be allowed.

1. Let p, q be twin primes, that is, primes satisfying $q = p + 2$. Prove that there is an integer a such that $p|a^2 - q$ if and only if there is an integer b such that $q|b^2 - p$.

2. Of which prime is -2 a quadratic residue.

3. For all prime p prove that $x^8 \equiv 16$ is solvable.(Hint : use the $a^{\frac{p-1}{(n,p-1)}} \equiv 1 \bmod p$ criterion in conjunction with the identity $16 = 2^4$).

4. Find all primes such that $x^2 \equiv 13 \bmod p$ has a solution. (Hint:use the reciprocity law).

5. If a is a quadratic nonresidue of each of the odd primes p, q, $x^2 = a \bmod ab$ solvable.

6. Show that if $m|d$ then $\phi(m)|\phi(d)$.

7. Show that if $a^3 \equiv 1 \bmod p$ then $1+a+a^2 \equiv 0 \bmod p$ and $(1+a)^6 \equiv 1 \bmod p$.

8. Show that if $p|\phi(d)$ and $p \nmid d$ (p is prime) then there is at least one prime factor q of d such that $q \equiv 1 \bmod p$.