Practice Midterm I

The actual midterm will consist of four problems.

Calculators will be allowed.

1. Show that that any positive integer of the form $4k+3$ has a prime factor of the same form.

2. Show that n^4+1 is composite for all $n>1$.

3. If n is composite, prove that $(n-1)!+1$ is not a power of n. (Hint: use mod n considerations).

4. Find the smallest positive integer giving remainders $1,2,3,4,5$ when divided by $3,5,7,9$ and 11.

5. Prove that any integer n can be expressed in the form $n=x^2+y^2-z^2$.

6. Find all rational points on the ellipse $4x^2+3y^2=1$.

7. Recall that $\mathbb{Z}_n$ is the group of residues mod n with the operation of addition. Find all homomorphisms of groups:

   a)from $\mathbb{Z}_4$ to $\mathbb{Z}_2 \times \mathbb{Z}_3$.

   b)from $\mathbb{Z}_2$ to $\mathbb{Z}_3^{\times}$- the group of invertible elements of the ring $\mathbb{Z}_3$ with the operation of multiplication.

8. Find all solutions of the equation $234x+567y=9$.