MAT211 Fall 2008

Practice Midterm I

Problem 1.

Consider the vectors $\vec{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

1. Find $\vec{v} + \vec{x}$, $\vec{v} - \vec{x}$, $\vec{x} - \vec{v}$, $proj_L(\vec{x})$ and $ref_L(\vec{x})$, where $L = span\{\vec{v}\}$. $proj_L$ and ref_L are projection and reflection related to L.

2. Represent graphically the vectors found in 1.

Problem 2. Consider a linear system $A\vec{x} = \vec{b}$ where A is a $m \times n$ matrix. You are told that this linear system always has a solution, no matter which $\vec{b}$ you choose.

1. Can you conclude that A is invertible?

2. What can you say about the kernel of A?

Problem 3. Consider the linear transformation $T : \mathbb{R}^6 \to \mathbb{R}^4$ given by $T(\vec{x}) = A\vec{x}$, where

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

1. Find bases for the image and the kernel of T.

2. Can you find three linearly independent vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ such that $T(\vec{v}_1) = \vec{0}, T(\vec{v}_2) = \vec{0}, T(\vec{v}_3) = \vec{0}$?

3. Does the linear system $A\vec{x} = \vec{b}$ have a solution for every $\vec{b} \in \mathbb{R}^4$? If not,for which vectors $\vec{b} \in \mathbb{R}^4$ does it have a solution?

Problem 4. Consider the set of vectors.

$$\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

1. Verify that $\mathfrak{B}$ is a basis for $\mathbb{R}^3$.

2. Write $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ in the basis $\mathfrak{B}$, i.e., find the components of $[\vec{x}]_{\mathfrak{B}}$

Problem 5. Find the matrix B of the linear transformation $T(\vec{x}) = A\vec{x}$ with respect to the basis $\mathfrak{B} =< \vec{v}_1, \vec{v}_2 >$ in two ways

1. As $B = S^{-1}AS$

2. Column by column.

The matrix A and the basis $\mathfrak{B}$ are

$$A = \begin{bmatrix} -3 & 4 \\ 4 & 3 \end{bmatrix}$$

$$\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$$

Problem 6.

1. What are the possible dimensions of linear subspaces in $\mathbb{R}^4$

2. Give examples of subspaces in $\mathbb{R}^4$ of each dimension found in (1)

3. Give five examples of subsets in $\mathbb{R}^4$ that are not subspaces.