MAT211 Fall 2008

Practice Final

The actual final will consist of nine problems(no subproblems)

You will be allowed to use calculators

Problem 1

Ch.2 Linear Transformations

Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

1. Find $3A$.
2. Find A^2 .
3. Find $A + I$.
4. Find A^T .
5. Find all pairs of columns which are perpendicular to each other.
6. Find which columns of A are parallel to each other.
7. Find all solutions to $A\mathbf{x} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$.
8. Is A invertible? Why or why not?
9. Is $A + I$ invertible? Why or why not?
10. What is A^6 ?

Problem 2.

Ch3. Subspaces of $\mathbb{R}^n$ and Their Dimensions

1. Let
$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$
Find bases for the image and kernel of A.

2. Let $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ the columns of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Define $T(\mathbf{x}) = A\mathbf{x}$. Find the matrix of T relative to the basis $\mathbf{v}_1 + \mathbf{v}_2$, $\mathbf{v}_2 + \mathbf{v}_3$, $\mathbf{v}_3 + \mathbf{v}_1$.

3. Let P be the vector space of all polynomials $p(x)$. Let S be the subset of P defined by
$$p(1) = p(0) + \int_0^1 xp(x)dx, \quad p(0.5) = 0.$$
Prove that S is a subspace of P.

Problem 3.

Ch.4 Linear spaces

Let $S = \{t^2 + 2t, t^2 - 4, t + 2\}$.

1. Is S a linearly independent set?
2. Is S a basis for P_3 , the set of all polynomials of degree three?
3. Is $2t^2 + 6t + 4$ in the span of the vectors in S?
4. Let $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. Let W be the linear space of all 3×3 matrices. Let V be the set of all 3×3 matrices A such that $\mathbf{x}$ belongs to the image of A. Prove or disprove: V is a subspace of W.
5. Let P_2 be the linear space of all quadratic functions $f(x) = c_0 + c_1 x + c_2 x^2$. Define $T(f) = c_2 x^2$ from P_2 to P_2 . Find the image, kernel, rank and nullity of T.
6. Let $\mathbb{R}^{4\times4}$ be the linear space of all real 4×4 matrices M . Let T be defined on $\mathbb{R}^{4\times4}$ by $T(M) = N$ where $N = M$ except for the last row, which is all zeros. Find the image and kernel of T.
7. Let $A = \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix}$, $S = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Then $AS = SD$. Define V to be the linear space of all 2×2 matrices R satisfying $AR = RD$. Find a basis for V.

Problem 4

Ch6. Orthogonality and Least Squares

1. Find the orthogonal projection of $\mathbf{v}$ onto $V = span(\mathbf{v}_1, \mathbf{v}_2)$, given

$$\mathbf{v} = \begin{pmatrix} 9 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{v}_1 = \begin{pmatrix} 0 \\ 4 \\ -1 \\ 0 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 0 \\ 4 \\ 1 \\ 0 \end{pmatrix}.$$

2. Let A be 4×5. Prove or give a counterexample: $\dim \operatorname{Im}(A)^{\perp} = \dim \operatorname{Ker} A^T$.

3. Let A be $n \times m$. Prove or give a counterexample: $\operatorname{Ker}(A) = \operatorname{Ker} AA^T$.

4. Find the Gram-Schmidt orthonormal vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ for the following independent set:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}.$$

5. Find the QR-factorization of $A = \begin{pmatrix} 4 & 10 & 0 \\ 0 & 0 & -1 \\ 3 & -10 & 0 \end{pmatrix}$

Problem 5

Ch.6 Determinants

1. Let B be the invertible matrix given below.

$$B = \begin{pmatrix} 7 & -6 & 2 & 0 \\ 0 & -1 & 2 & 0 \\ 1 & 1 & 0 & 0 \\ 2007 & 2008 & 2009 & 3 \end{pmatrix}$$

   Find the value of $\det(2B^{1}(B^{T})^{2})$.

2. Find the area of the parallelogram formed by $\mathbf{v}_1, \mathbf{v}_2$, given

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

3. Evaluate $\det(A)$ by any hybrid method.

$$A = \begin{pmatrix} 1 & 1 & 2 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & 2 & 1 & 0 \\ 1 & 1 & 2 & -3 \end{pmatrix}$$