10/20/08.

3.3.

6. $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \end{bmatrix} = A$

the rank $A = 2$, Hence $\operatorname{im} A = \mathbb{R}^2$, and by observation

$\ker A = \{ [1 \ 2 \ -1]^T \}$

39. $A_{5\times5} = B_{5\times4} \, C_{4\times5}$

Notice that $\ker A_{5\times5} \subset \ker C_{4\times5}$

but $\dim \ker C_{4\times5} + \dim \operatorname{im} C_{4\times5} = 5$

which means $\dim \operatorname{im} C_{4\times5} \leq 4$, then $\dim \ker C_{4\times5} \geq 1$

Hence $\dim \ker A \geq 1$, Hence non-invertible.

3.4.

2. $\vec{x} = \begin{bmatrix} 3 \\ -4 \end{bmatrix} = -4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \left[-4\vec{v}_1 + 3\vec{v}_2 \right] = \begin{bmatrix} -4 \\ 3 \end{bmatrix}_B$

4. $\vec{x} = \begin{bmatrix} 23 \\ 29 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 46 \\ 58 \end{bmatrix} + 0 \cdot \begin{bmatrix} 61 \\ 67 \end{bmatrix} = \frac{1}{2} \vec{v}_1 = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}_B$

6. $\vec{x} = \begin{bmatrix} -4 \\ 4 \end{bmatrix} = 11 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 3 \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 11 \\ -3 \end{bmatrix}_B$

20. $S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $S^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$B = S^{-1}AS = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

Other situations are the same. but should be mentioned.