MAT 530

Topology I

Lecturer: Mikhail Lyubich
Grader: Ilya Elson

This class will give an introduction to basic concepts of topology, which lay down a foundation for most of mathematics: analysis, differential and algebraic geometry, dynamical systems theory,... It will consists of two parts: basic general topology (topological and metric spaces, connectivity and compactness, function spaces,..., with fractal geometry illustrations) and first glance into algebraic topology (fundamental group, covering spaces, classification of surfaces,...)

Lectures: TuTh 11:20 – 12:40, Math Build, room 5-127.

Textbook: J. Munkres. Topoloigy.

The final grade will be based upon HW (25%), Midterm (35%) and the Final (40%).

Homeworks:

HW1 (due Sept 8): p. 92 (# 3,6), p. 101 (# 6,13), p. 102 (# 20), p. 112 (# 9,12), p. 144-5 (# 2,4).

HW2 (due Sept 15): p.127 # 4 (functions f and h, sequences w and y), # 5,6,8; p. 134 (6,8,9,10), p. 146 (2(e), 5,6)

HW3 (due Sept 20): p. 135, # 11; p. 152. # 1,2,6, 9, 10, 11.
Bonus problem: p. 102, # 21.

HW4 (due Sept 27): pp. 157-158, # 1,2,3, 11; p. 162, # 2 (a,b), 6, 7, 8, 9.

HW5 (due Oct 4): pp. 170-171, # 1, 4, 5, 7, 8, 10(a), 11, 12.

HW6 (due Oct 11): p. 178, # 4,5, p. 181, # 6,7, p. 280, # 7, 275, # 4(a) (except metrizability).

Bonus: Prove that the Koch snowflake is a Jordan curve (injectivity!).

HW8 (due Oct 25): p. 270, # 1, 2, 3, 5, 8; p. 299, # 7,8,9.

Bonus: Consider the function $\phi(x)$ equal to $|x|$ on $[-1, 1]$ and then periodically extended to the whole real line. Prove that the function

$$f(x) = \frac{1}{2^n}\phi(2^n x)$$

is continuous but nowhere differentiable.

HW9 (due Nov 1): p. 292, # 1, 5.

A function $f : X \to \mathbb{R}$ is called *upper semincontinuous* at $x_0 \in X$ if for any $\epsilon > 0$ there exists a neighborhood $U \ni x$ such that $f(x) < f(x_0) + \epsilon$ for all $x \in U$.
1) Show that f is semi-continuous (at all points) if and only if preimages of the rays $(-\infty, a)$ are open;
2) Show that the set of zeros of an upper semi-continuous function $f \geq 0$ has type G_δ (countable intersection of open sets).

HW10 (due Nov 8): p. 330, # 1–3; p. 335, # 3–5.

A space is called *separable* if it contains a dense countable subset.
1) Show that 2nd Countability Axiom implies separability.
2) Show the inverse for metrizable spaces.
3) Show that a metrizable compact space is separable.

HW11 (due Nov 15): p. 341, # 2–4, 6 (skip completely regular case); p. 347, # 1, 4, 5.

HW12 (due Nov 22): p. 356, # 1,2; p.366, # 2,4,6,9.

HW13 (due Nov 29): p.367, # 10; p. 370, # 1–4.

HW14 p. 375, # 2, p. 483, # 1,2,4,5 (due Dec 6); p. 454, # 3–7 (due Dec 8).

HW15 (due Dec 13) p. 492, # 1,2(a,b), 5; p. 483, # 1,2,4. (More may come)

Office hours: Tue 10–11 a.m., Thu 2:30–3:30 p.m.

Midterm: Thu, October 13 during the class